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MHD Forced Convective Flow of Micropolar Fluids Past a Moving Boundary Surface with Prescribed Heat Flux and Radiation

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Authors' contributions

This work has been carried out in collaboration between all authors. Author HW performed mathematical analysis and computational work. Author SH designed the study. Authors HS and SK prepared introduction, reference, results and discussion sections and typing/ formatting of the article. All authors read and approved the final manuscript.

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Original Research Article

Abstract

The forced convective boundary layer flow of electrically conducting micropolar fluids has been investigated in the presence of magnetic field applied in the normal direction of a sheet that shrinks or stretches horizontally and thus causes the fluid motion. Self-similar transforms have been employed to convert the governing partial differential equations into ordinary differential form. The resulting highly non-linear model has been solved numerically with coding in Mathematica. Rigorous computational work has been carried out for sufficient ranges of the parameters of the study namely suction parameter *S*, stretching/shrinking parameter \in , magnetic parameter *M*, material constants d_1 , d_2 , d_3 involved in micromotion, heat source parameter *B*, radiation parameter R_n , heat flux parameter *n* and Prandtl number *Pr*. The effects of these parameters on the physical quantities like skin friction coefficient, velocity, temperature and micromotion are presented graphically.

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1 Introduction

The micropolar fluid theory pioneered by Eringen [1] presents relatively an interesting and applied research field. This model besides the generalization of Navier-Stokes model takes into account the conservation of angular momentum due to local micromotion of the fluid particles. Researchers are engaged to explore innovative results related to micropolar fluids flow problems. Heat transfer in micropolar fluids was discussed by Eringen himself [2]. Kasiviswanathan and Gandhi [3] studied the magnetohydrodynamic flow of a micropolar fluid and obtained the exact solution. Magyari et al. [4] considered the flow of micropolar fluids and solved problem by Laplace transformation. Kelson and Farrell [5] analyzed self-similar boundary layer flow of a micropolar fluid in a porous channel where the flow was driven by uniform mass transfer through the channel walls. Haque et al. [6] worked on micropolar fluid behavior with steady magnetohyrodynamic free convention and mass transfer flow. Kishore et al. [7] examined the magneto hydrodynamic heat and mass transfer and thermal radiation, with viscous dissipation effects on magnetohyrodynamic flow of micropolar fluid. Khana et al. [8] investigated the effects of heat transfer on a peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field.

Rashidi et al. [9] examined the steady, incompressible and laminar flow of micropolar fluids inside an infinite channel. The steady two- dimensional viscous incompressible stagnation point flow on moving boundary sheet was investigated by Mahapatra and Gupta [10]. Bakr et al. [11] studied the effects of chemical reaction and thermal radiation on unsteady free convection flow of a micropolar fluid past a semiinfinite vertical plate embedded in a porous medium in the presence of heat absorption with Newtonian heating. The flow and heat transfer phenomenon in a power law fluid over a porous stretching sheet with effect of magnetic field was considered by researchers mentioned in [12–13]. For effects of the buoyancy force and thermal radiation in MHD boundary layer viscoelastic fluid over continuously moving stretching surface in a porous surface see [14-15]. Raptis [16] studied the flow of a micropolar fluid past a continuously moving plate in the presence of radiation. Cortell [17] studied the effects of viscous dissipation and radiation on the thermal boundary layer over a non-linear stretching sheet.

The thermal radiation and heat generation effects on MHD convective flow are a new dimension added to the study of stretching surface and has important applications in physics and engineering particularly in space technology and high temperature processes. Recently, Hassan et al. [18] worked on mathematical analysis and numerical solution for micropolar fluids flow due to a shrinking porous surface in the presence of magnetic field and thermal radiation. Ajaz and Elangovan [19] studied influence of an inclined magnetic field and rotation on the peristaltic flow of a micropolar fluid in an inclined channel. The effect of radiation on heat transfer problem was studied by Hossain and Takhar [20]. Hassan et al. [21] studied the micropolar fluids near the stagnation point flow of electrically conducting fluid due to a surface with the boundary in motion (stretching/shrinking). Ajaz and Elangovan [22] presented theoretic study on the electro-osmotic flow of a micropolar fluid in a porous microchannel and considered the effects of the inclined magnetic field and electro-osmotic parameters on the kinematics of the fluid.

This paper considers forced convective boundary layer flow of micropolar fluids past a stretching or shrinking sheet in the presence of prescribed heat flux and heat source with radiative effect. Anjali and Raj [23] studied this problem for Newtonian fluids, without radiation. We made a comprehensive mathematical and computational analysis of the problem to examine the flow, micromotion and heat transfer characteristics.

2 Mathematical Model

We assume incompressible micropolar fluid with steady, two dimensional, laminar hydromagnetic boundary layer flow due to stretching/shrinking surface. A uniform magnetic field of strength B_0 is applied in the direction parallel to y-axis normal to the sheet lying horizontally along x- axis. The pressure gradient, the body couple, induced magnetic field and viscous dissipation are negligible. The velocity vector is $\underline{V} = V(u, v)$ and spin vector is $\underline{\omega} = \omega(0, 0, \omega_3)$ and fluid temperature is *T*, where $T_w(x)$ is temperature at the boundary of the surface and q_r is radiative heat flux. The schematic diagram of fluid flow is depicted in Fig. 1.

Fig. 1. Sketch of flow model

Under the above assumptions, the equations governing the problem (Eringen [1], Anjali and Raj [23]) become:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$
 (1)

$$
(\mu + k) \frac{\partial^2 u}{\partial y^2} + k \frac{\partial \omega_3}{\partial y} - \sigma B_0^2(x)u = \rho(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y})
$$
\n(2)

$$
\gamma \left(\frac{\partial^2 \omega_3}{\partial y^2} \right) - \kappa \left(\frac{\partial u}{\partial y} + 2\omega_3 \right) = \rho \, j \left(u \, \frac{\partial \omega_3}{\partial x} + v \, \frac{\partial \omega_3}{\partial y} \right) \tag{3}
$$

$$
u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0 (T - T_\infty)}{\rho C_p} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y}
$$
(4)

where ρ is density, σ is the electrical conductivity, C_p is the specific heat capacity at constant pressure, μ is dynamic viscosity, *k* and γ are additional viscosity coefficients for micropolar fluid and *j* is micro inertia , *K* is the thermal conductivity, $Q_0(T - T_{\infty})$ is heat generated or absorbed per unit volume, B_0 is the applied magnetic induction, the radiation term $a = -\frac{4\sigma^*}{r^4}\frac{\partial T^4}{\partial t^4}$ $q_r = -\frac{4\sigma^*}{3\beta} \frac{\partial T}{\partial y}$ σ β $=-\frac{4\sigma^*}{3\beta}\frac{\partial \Omega}{\partial}$ $,k^*$ is as mean absorption coefficient, σ^* Stefan

Boltzmann constant in the thermal radiation, β is mean absorption coefficient. With the assumption that T^4 is expanded in Taylor series about T_{∞} and neglecting higher order terms to get $T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4$.

The boundary conditions are:

$$
-k\frac{\partial T}{\partial y} = q_W = Dx^n, \ \omega_3 = -m\frac{\partial u}{\partial y}, \ u = bx, \ v = -v_0, \ \text{when} \ \ y \to 0
$$

$$
\omega_3 \to 0, \ u \to 0, T \to T_\infty \quad \text{as} \ \ y \to \infty
$$
 (5)

where, v_0 is the suction/injection velocity and *m* is a boundary parameter($0 \le m \le 1$) that is used to model the extent to which microelements are free to rotate in the vicinity of the sheet. We assumed that microelements do not rotate at the boundary (*m*=0).

Using similarity transformations:

The velocity components are described in terms of the stream function $\psi(x, y)$

$$
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \psi(x, y) = x\sqrt{av} f(\eta), \quad \eta = y\sqrt{\frac{a}{v}}
$$

$$
u = xaf', \quad v = -\sqrt{var} f, \quad \omega_3 = \frac{a^{\frac{3}{2}}}{v^{\frac{1}{2}}} xL(\eta), \quad T - T_{\infty} = \frac{Dx^n}{K} \sqrt{\frac{v}{a}} \theta(\eta),
$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4), we get

$$
(1+d_1)f''' + d_1L' - Mf' = f'^2 - ff''
$$
\n(6)

$$
d_{3}L'' + 2 d_{1}d_{2}L - d_{1}d_{2}f'' = f L - f L'
$$
\n(7)

$$
(4+3Rn)\theta'+3Rn \Pr(f\theta'-nf'\theta+B\theta)=0
$$
\n(8)

and the boundary conditions are

$$
f'(0) = \varepsilon, f(0) = S, L(0) = 0, \theta'(0) = -1,
$$

$$
f'(\infty) = 0, L(\infty) = 0, \theta(\infty) = 0,
$$
 (9)

whereas
$$
M = \left(\frac{\sigma}{\rho a}\right) B_0^2
$$
 is the magnetic parameter, $P_r = \frac{\mu C_p}{K}$ is Prandtl number, $R_n = \frac{16\sigma^* T^3}{3\beta K}$ is

thermal radiation parameter, $S = \frac{v_0}{\sqrt{2}}$ *a*^υ $=\frac{v_0}{\sqrt{2}}$ is the suction/injection parameter, $B=\frac{Q_0}{Q_0}$ *p Q C B a*ρ $=\frac{20}{10}$ is heat source

parameter and * *p* $n = \frac{\beta^* x}{\alpha}$ *C* β ρ $=\frac{p}{q}$ is heat flux parameter and $\epsilon = \frac{b}{q}$ $\frac{b}{a}$ is the velocity ratio parameter, $\epsilon < 0$ denotes

shrinking sheet, ϵ > 0 is for stretching sheet and *K* is thermal conductivity.

The dimensionless material constants are:

$$
d_1 = \frac{k}{\mu}, d_2 = \frac{\mu}{\rho \, j \, a}, d_3 = \frac{\gamma}{\rho \, j \, v}
$$

3 Results and Discussion

The numerical solution for the set of coupled non-linear differential equation (6) to (9) has been attempted because their analytical solution is difficult to find. For the numerical purpose, the order of these equations is reduced by letting $f' = p$, $p' = q$, $L' = g$, $\theta' = w$. The resulting set of ODE's is of first order which is then solved numerically using codes in Mathematica 10. Results have been computed for sufficient ranges of the parameters of interest. The fixed values of the parameters are chosen arbitrarily as $S = 3, M = 2, B = 0.05,$ \therefore Pr = 0.71, $\varepsilon = -1$, $n = 2, d_1 = 0.5, \therefore d_2 = 1.5, d_3 = 2$. In particular, the results for material parameters d_1, d_2, d_3 describe the micropolar nature of the fluid. When $d_1 = 0$ and *L*=0, the flow pattern becomes same as for Newtonian fluids. The results have been presented in the form of tables for skin friction coefficient *f* $\binom{n}{0}$ and initial boundary values of $\theta(\eta)$ and in the form of plots for velocity *f* ', temperature θ and micromotion *L*. Tables 1 and 2 respectively demonstrate the results for *f* "(0) and θ (0). Both of these tables show, a good comparison of present results with the previous results by Anjali and Raj [23] and hence the validation of present results. Moreover, it is noticed that values of $f''(0)$ for micropolar fluids are less than the corresponding values $f''(0)$. Also, the values of $\theta(0)$ are larger in magnitude in the presence of radiative heat source. Figs. 2 and 3 respectively demonstrate the effect of magnetic force field and suction at the boundary. Both the physical parameters decrease in the magnitude of the fluid flow velocity *f* ' when the surface is shrinking. But the velocity magnitude increases with increase in the values of micropolar parameter d_1 and shrinking parameter ϵ (ϵ < 0) as shown in Figs. 4 and 5 respectively.

Fig. 6 shows the effect of stretching parameter ϵ (ϵ > 0) on velocity f' , a sufficient increase in the velocity is observed with increase in the values of parameter*є* . The graph for vertical velocity *f* under the effect of *S* is presented in Fig.7.

The non-dimensional micromotion function *L* has been mapped in Fig. 8 to indicate the effect of stretching/shrinking parameter*є*. The micromotion increase in magnitude near the surface but opposite behavior is seen away from the surface. Fig. 9 shows that micromotion increases with increase in magnetic force field. The effect of micropolar parameter d_1 on micromotion *L* is demonstrated in Fig. 10. The micromotion *L* increases with increase in the values of d_1 .

Figs. 11 and 12 respectively shows the effect of heat source parameter *B* and heat flux parameter *n* on temperature function $\theta[\eta]$. Both these parameters cause an increase in temperature function $\theta[\eta]$. But increase in magnetic force causes decrease $\theta[\eta]$ as shown in Figs. 13. 14 shows that function $\theta[\eta]$ decreases when the sheet is stretching and it is opposite to the shrinking phenomenon. Suction at the surface and Prandtl number P_r cause decrease in temperature function $\theta[\eta]$, as shown in Figs. 15 and 16 respectively. But the radiation parameter R ^{*n*} causes increase in temperature distribution $\theta[\eta]$ as depicted in Fig. 17. The results are in accordance with the physical nature of the pertinent parameters.

Anjali and Raj [23]	Present results	Present results	Parameters	Anjali and Raj [23]	Present results	Present results	Parameters
	(Newtonian) fluids)	(Micropolar fluid)			(Newtonian) fluids)	(Micropolar fluid)	
2.414214	2.41448	2.22547	$S = 2$	3.302775	3.30281	3.02707	P_{r} = 1
3.302775	3.30304	3.02707	$S = 3$	3.302775	3.30281	3.02707	$B=0$
4.236068	4.23607	3.87095	$S = 4$	3.302775	3.30281	3.02707	$B = 0.05$
5.192583	5.19258	4.73721	$S = 5$	3.302775	3.30281	3.02707	$n = -2$
6.162278	6.16228	5.61632	$S = 6$	3.302775	3.30281	3.02707	$n = -1$
2.618034	2.61941	2.34215	$M = 0$	3.302775	3.30281	3.02707	$n=0$
3.000000	3.00019	2.72747	$M = 1$	3.302775	3.30281	3.02707	$n=1$
3.302775	3.32281	3.02707	$M = 2$	1.718246	1.71826	1.57922	$\mathcal{E} = 0.5$
3.561552	3.56156	3.28064	$M = 3$	-1.83972	-1.83973	-1.69794	$\mathcal{E} = 0.5$
3.791287	3.79129	3.50446	$M = 4$	-3.79128	-3.7913	-3.50476	$\mathcal{E}=1$

Table 1. Comparison of results for skin friction coefficient *f* ''(0)

Fig. 2. The plot for curves of f' under the effect of magnetic parameter M

Fig. 3. The plot for curves of *f* ′ **under the effect of suction/injection parameter S**

Fig. 4. The plot for curves of f' under the effect of micropolar parameter d_1

Fig. 5. The plot for curves of *f* ′ **under the effect of shrinking parameter** ∈

Fig. 6. The plot for curves of f' under the effect of stretching parameter \in

Fig. 7. The plot for curves of *f* **under the effect of suction/injection parameter** *S*

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Fig. 8. The plot for curves of *L* **under the effect of stretching/shrinking parameter** ∈

Fig. 9. The plot for curves of *L* **under the effect of magnetic parameter** *M*

Fig. 10. The plot for curves of L under the effect of Micropolar parameter d_1

Fig. 11. The plot for curves of θ under the effect of heat source parameter B

Fig. 12. The plot for curves of θ **under the effect of heat flux parameter** n

Fig. 13. The plot for curves of θ under the effect of magnetic parameter M

Fig. 14. The plot for curves of *θ* **under the effect of stretching/shrinking parameter** ∈

Fig. 15. The plot for curves of *θ* **under the effect of suction/injection S**

Fig. 16. The plot for curves of θ **under the effect of Prandtl number** P_r

Fig. 17. The plot for curves of θ **under the effect of radiation parameter** R_n

4 Conclusion

This work has been carried out to examine the of magneto hydrodynamic boundary layer flow and heat transfer for micropolar fluids, owing to permeable sheet that shrinks or stretches linearly. The parametric study of the problem involving a reliable computational technique produced several useful results that are summarized as below:

- Magnetic force field and suction at the boundary, both decrease velocity f' when the surface is shrinking.
- The velocity magnitude increases with increase in the values of micropolar parameter d_1 and shrinking parameter ϵ (ϵ < 0).
- The stretching parameter ϵ ($\epsilon > 0$) causes, a sufficient increase in the velocity f' .
- The micromotion increases with increase in magnetic force field and micropolar parameter d_1 .
- The effect of heat source parameter B and heat flux parameter n , both cause an increase in temperature function $\theta[\eta]$.
- The increase in magnetic force causes decrease in $\theta[\eta]$.
- The function $\theta[\eta]$ decreases when the sheet is stretching but opposite effect is observed for shrinking sheet.
- Suction at the surface and Prandtl number P_r cause increase in $\theta[\eta]$.
- The radiation parameter R_n causes increase in temperature distribution $\theta[\eta]$.

Competing Interests

Authors have declared that no competing interests exist.

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