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Superluminal Hydrogen Atom in a Constant Magnetic Field in (3+1)-dimensional Spacetime (I)

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

In this paper, we have checked Stern-Gerlach experiment with the aim to study generic effect of an applied magnetic field onto transversely directed beam of hydrogen-like atoms. The ultrarelativistic phenomenon of spin of a Dirac particle (especially, electron spin) producing a continuum of linear angular momentum with the known result of superluminal propagation, suggests the feasibility of similar dynamics for a charged hydrogen-like particle under applied magnetic field, in spacetime. Another mechanism, very important but popularly less comprehensible, which sustains this linear momentum is known to be helical plane wave expansion. Hydrogen-like spherical waves cannot perform this function due to the perturbation caused by the successive random orientations of their atomic magnetic moment. It is therefore of vital import to investigate experimentally, as well as analytically, the possibility of transformation from hydrogen-like spherical wave expansion to its probable plane wave function, if we would extend our special subatomic theory of superluminal particles to the atomic (hydrogen-like) level.

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1 INTRODUCTION

"Stern-Gerlach Experiment (SGE) or Transformation from Spherical Wave Expansion into Plane Wave Functions". This could perfectly fit in as the title for this article. But the implication of the SGE application, or in general of the effect of an applied magnetic field on a transversely guided beam of hydrogen-like atoms in ground state, which we prove (in the second paper of this series) to result in excess of the speed of light, is so vitally huge that it eclipses the above function-label title put in quotation marks. Multiple influential functional aspects have been uncovered about this experiment from its first exhibition in 1922 to date, and they include space quantization, angular momentum and directional quantization, measurement (for the first time) of a ground state property of an atom and production of a fully "spin-polarized" atomic beam, and manifestation of the spin of the electron. This list could not have ended there if the phenomenon of spin were more comprehensible to the scientific elite. Though Einstein and Ehrenfest [1] [2], Heisenberg [3] [4], Phipps and Stern [5], and others historically made early serious attempts, through physical processes, to demonstrate directional quantization in the SGE, not much progress has been accomplished since, if not controversies, rather.

It is worthwhile to better comprehend the following physical (realistic) entities: spherical wave, plane wave, magnetic moment, angular momentum, spacial rotations and their relations to the spin "non-realistic" phenomenon. Firstly and most importantly, we must understand that spherical waves do not and cannot produce a continuum of linear angular momentum, simply because of the system perturbation caused by the successive and fast random orientations of the hydrogen-like atomic nuclear magnetic moment which is co-linear with the angular momentum vector. A linear angular momentum - our interest in this work and as exhibited in the SGE (in spin-up and spin-down angular momentum) - is always underpinned and can only be sustained by a flow of helical plane waves [6] [7] [8]. Secondly, it had not been an easy task for the pioneers to reconcile the "non-realistic" quantum theory and the actual physical theory of the universe, namely special/general relativity, for which cause emerged the joint-theory of relativistic quantum field (with the related topics thereof) [9] [10]. Especially, it is no small matter to link or establish a mathematical relationship between the subatomic non-realistic spin phenomenon and the reality represented by spacial rotations; however, based on the geometry of Minkowski spacetime [11], using the *spinor map*, we have been able to derive a definite formula which is simple and predictable but somewhat cumbersome to obtain, while at the time we could neither imagine nor guess for an inch how seminal our effort could be; one can check and find the details in reference [12]. As Louis Pasteur puts it, in science a situation of coincidence only favours or makes things easier for minds who have been prepared.

In the light of all these considerations, it should by now appear plausible why we say it is not at all a trivial matter to discern and demonstrate that the SGE performs a transformation from spherical wave expansion into hydrogen-like helical plane wave functions, in spacetime. We will revisit the SGE in order to determine the analytical transformation which supports our claim, using our result in reference [12]. In Section 2 we briefly review the mathematics behind the derivation of hydrogen-like spherical wave functions in the context of Dirac relativistic wave equation. We show in Section 3 that there exists a class of spherical harmonic functions whose quantum numbers l, k, and m satisfy a certain mathematical relation, which brings to feasibility the transformation.

2 THE RELATIVISTIC HYDROGEN-LIKE SPHERICAL WAVE FUNCTIONS

In a limited arena such as this paper, we will restrict our considerations to our main interest, the spherical harmonics, which play an essential part in the hydrogen spherical wave functions, while holding a summary view on the remaining ingredients, employing a combined approach [13] [14] [15].

Let us consider the hydrogen atom in the context of relativistic quantum theory using the Dirac equation [9]. We regard the hydrogen atom as one-particle system of reduced mass m, where the electron orbits a dense and static nucleus with spin in a central potential $V(r) = -e^2/r$, bringing the case to a Coulomb problem. The Dirac equation which describes it is

$$H\psi = [H_D + V(r)]\psi = \varepsilon\psi, \tag{2.1}$$

where ψ, the eigenvalue ε, and H_D denote possible states of the electron, the electron energy, and the Dirac Hamiltonian, respectively.

$$H_D = c\boldsymbol{\alpha}\cdot\boldsymbol{p} + mc^2\beta, \tag{2.2}$$

and, $\boldsymbol{\alpha}$ and β are the 4×4 Dirac matrices.

The total angular momentum is given by $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, where $\boldsymbol{L}$ is the orbital angular momentum, and $\boldsymbol{S} = (\hbar/2)\begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$, the 4×4 spin angular momentum matrix, with $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ being the Pauli matrices. It can readily be shown that $\boldsymbol{J}$ commutes with the Dirac Hamiltonian H_D, provided the potential V is isotropic. We may, therefore, classify eigenstates of H_D according to the eigenvalues of energy, $\boldsymbol{J}^2$ and J_z. Eigenstates of $\boldsymbol{J}^2$ and J_z are readily constructed using the two-component representation of $\boldsymbol{S}$. They are the spherical spinors defined below.

2.1 Spherical Harmonics and Spherical Spinors

One can express the orbitals of a given state in terms of two angular functions and two radial functions of arguments the principal quantum number n, a quantum number m, and an integer k, defined by

$$k = \{^{-j-\frac{1}{2} \quad if \quad j=l+\frac{1}{2}}_{j+\frac{1}{2} \quad if \quad j=l-\frac{1}{2}}, \qquad -j/2 \le m \le j/2 \quad of\ step\ 1, \tag{2.3}$$

where l, with $0 \le l \le n-1$, denotes the azimuthal quantum number.

The expression of the energy, obtained as a function of n and $|k|$ (for $|k| = j + 1/2$), is based on the ground state (or zero-point) energy and given as

$$E_{n,j} = \mu c^2 \left[1 + \left(\frac{Z\alpha}{n - |k| + \sqrt{k^2 - \alpha^2}}\right)^2\right]^{-1/2}, \tag{2.4}$$

where μ is the reduced electron-proton mass, Z the atomic number (i.e., the number of protons) and $\alpha \simeq 1/37$ is the *fine structure constant,* which is a deterministic modification factor between the energy solutions in the Dirac and Schrödinger equations.

Using the two-component spinors $\chi_{\mu=\pm 1/2}$:

$$\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

we can classify the solution to the Dirac-hydrogen equation for quantum numbers n, k, and m, as

$$\Psi_{nkm} = \begin{pmatrix} g_{n,k}(r)\, r^{-1}\Omega_{k,m}(\theta,\phi) \\ if_{n,k}(r)\, r^{-1}\Omega_{-k,m}(\theta,\phi) \end{pmatrix} = \begin{pmatrix} g_{n,k}(r)\, r^{-1}\sqrt{\frac{k+\frac{1}{2}-m}{2k+1}}Y_{k,m-1/2}(\theta,\phi) \\ -g_{n,k}(r)\, r^{-1} sgn\, k\sqrt{\frac{k+\frac{1}{2}+m}{2k+1}}Y_{k,m+1/2}(\theta,\phi) \\ if_{n,k}(r)\, r^{-1}\sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}}Y_{-k,m-1/2}(\theta,\phi) \\ -if_{n,k}(r)\, r^{-1} sgn\, k\sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}}Y_{-k,m+1/2}(\theta,\phi) \end{pmatrix}, \tag{2.5}$$

where $\Omega_{\pm k,m}$, known as spherical spinors, are the two-component spinors formed by the entries involving the spherical harmonic functions $Y_{\pm k,m\mp 1/2}$, shown to the right, which in turn are defined as follows

$$Y_{a,b}\left(\theta,\phi\right)=\{^{(-1)^b\sqrt{\frac{2a+1}{4\pi}\frac{(a-b)!}{(a+b)!}}P_a^b(\cos\theta)e^{ib\phi}\quad if\quad a>0}_{Y_{-a-1,b}(\theta,\phi)\qquad\qquad\qquad if\quad a<0.} \tag{2.6}$$

Here P_a^b is an associated Legendre polynomial in $x=\cos\theta$, which is a canonical solutions of the general Legendre equation

$$\left(1-x^2\right)\frac{d^2}{dx^2}P_l^m\left(x\right)-2x\frac{d}{dx}P_l^m\left(x\right)+\left[l\left(l+1\right)-\frac{m^2}{1-x^2}\right]P_l^m\left(x\right)=0. \tag{2.7}$$

In quantum mechanics, spherical harmonics obey the orthogonality condition

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}Y_l^mY_{l'}^{m'\star}d\Omega=\delta_{ll'}\delta_{mm'}, \tag{2.8}$$

where $\star$ stands for "complex conjugate" and $d\Omega=\sin\theta d\theta d\phi$, leading to the normalization condition

$$\int\left|Y_l^m\right|^2d\Omega=1 \tag{2.9}$$

in order to guaranty that probability is normalized, and where one must note that the notations Y_l^m and $Y_{l,m}$. are identically the same.

Under rotation transformation R about the origin of a given coordinate system which takes the unit vector $\boldsymbol{r}$ to $\boldsymbol{r}'$, a spherical harmonic of degree l and order m transforms into a linear combination of spherical harmonics of the same degree:

$$Y_l^m\left(\boldsymbol{r}'\right)=\sum_{m'=-l}^{l}\left[D_{mm'}^{(l)}\left(R\right)\right]^\star Y_l^{m'}\left(\boldsymbol{r}\right), \tag{2.10}$$

with $D_{mm'}^{(l)}\left(R\right)$ being a Wigner D-matrix of order $(2l+1)$ which depends on R. Equation (2.10) describes a group theoretical property (i.e., a rotational behaviour) of spherical harmonics. In particular, it can be shown that the functions Y_l^m of degree l form a complete orthonormal basis for the irreducible representation of the group $SO\left(3\right)$ of dimension $(2l+1)$. This symmetry property paves the way to bringing cumbersome solutions of some analytical problems on spherical harmonics (such as the Addition theorem) to simplified form.

The parity operator P maps $\boldsymbol{r}\to-\boldsymbol{r}$. In spherical coordinates, the operator P transforms $\phi\to\phi+\pi$ and $\theta\to\pi-\theta$. Under a parity transformation,

$$PY_{lm}\left(\theta,\phi\right)=Y_{lm}\left(\pi-\theta,\phi+\pi\right)=\left(-1\right)^lY_{lm}\left(\theta,\phi\right). \tag{2.11}$$

Hence the spherical spinors are eigenfunctions of P having eigenvalues $\pi=(-1)^l$. The two spinors $\Omega_{k,m}\left(\theta,\phi\right)$ and $\Omega_{-k,m}\left(\theta,\phi\right)$, corresponding to the same value of j, have values of l differing by one unit and, therefore, have opposite parity. Spherical spinors transform as

$$\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}\Omega_{k,m}\left(\theta,\phi\right)=\Omega_{-k,m}\left(\theta,\phi\right), \tag{2.12}$$

under the operator $\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}$, where $\hat{\boldsymbol{r}}=\boldsymbol{r}/r$.

Special cases and values of spherical harmonics are obtained in the following three points:

1. The case $m=0$ reduces these functions to ordinary Legendre polynomials

$$Y_l^0\left(\theta,\phi\right)=\sqrt{\frac{2l+1}{4\pi}}P_l\left(\cos\theta\right).$$

2. When $m = \pm l$,
$$Y_l^{\pm l}(\theta, \phi) = \frac{(\mp 1)^l}{2^l l!}\sqrt{\frac{(2l+1)!}{4\pi}}\left(\sin^l \theta\right) e^{\pm il\phi}$$
3. At the north pole where $\theta = 0$ and ϕ is undefined, all spherical harmonics for which $m \neq 0$ vanish, and we have
$$Y_l^m(0, \phi) = Y_l^m(z) = \sqrt{\frac{2l+1}{4\pi}}\delta_{m0}.$$

Thus, these cases show that a list of analytic expressions of spherical harmonics can be readily generated (at least for the first few terms) using a corresponding counter-list of the associated Legendre polynomials, a task, however, we will not undertake to establish in this so restricted arena.

Next, to write the functions $g_{n,k}(r)$ and $f_{n,k}(r)$ found in (2.5), we first define a constant γ and a scale radius ρ as follows:

$$\gamma \equiv \sqrt{k^2 - Z^2\alpha^2}, \qquad \rho \equiv 2Cr \qquad with \qquad C = \frac{\sqrt{\mu^2 c^4 - E^2}}{\hbar c}; \tag{2.13}$$

here the quantity E is the energy given in (2.4).

Where $k = -n$, which corresponds to the maximum possible $j-$value for a given n, then $g_{n,k}(r)$ and $f_{n,k}(r)$ become

$$\begin{aligned} g_{n,-n}(r) &= A(n+\gamma)\rho^\gamma e^{-\rho/2} \\ f_{n,-n}(r) &= AZ\alpha\rho^\gamma e^{-\rho/2}, \end{aligned} \tag{2.14}$$

with A being a normalization constant which depends on the Gamma function:

$$A = \frac{1}{\sqrt{2n(n+\gamma)}}\sqrt{\frac{C}{\gamma\Gamma(2\gamma)}}. \tag{2.15}$$

One must note that, in the limiting case where the negative energy part $f(r)$ is such that $f(r) \ll g(r)$ because of the factor $Z\alpha$, the energy E and the radial decay constant C become, respectively

$$E_{n,n-1/2} = \frac{\gamma}{n}\mu c^2 = \sqrt{1 - \frac{Z^2\alpha^2}{n^2}}\mu c^2 \qquad and \qquad C = \frac{Z\alpha}{n}\frac{\mu c^2}{\hbar c}. \tag{2.16}$$

Generally, when $k \neq -n$, $g_{n,k}(r)$ and $f_{n,k}(r)$ are expressed in terms of two generalized Laguerre polynomials L_a^b:

$$\begin{aligned} g_{n,k}(r) &= A\rho^\gamma e^{-\rho/2}\left[Z\alpha\rho L_{n-|k|-1}^{2\gamma+1}(\rho) + (\gamma - k)\frac{\gamma\mu c^2 - kE}{\hbar cC}L_{n-|k|}^{2\gamma-1}(\rho)\right] \\ f_{n,k}(r) &= A\rho^\gamma e^{-\rho/2}\left[(\gamma - k)\rho L_{n-|k|-1}^{2\gamma+1}(\rho) + Z\alpha\frac{\gamma\mu c^2 - kE}{\hbar cC}L_{n-|k|}^{2\gamma-1}(\rho)\right] \end{aligned} \tag{2.17}$$

where A now assumes the form

$$A = \frac{1}{\sqrt{2k(k-\gamma)}}\sqrt{\frac{C}{n-|k|+\gamma}\frac{(n-|k|-1)!}{\Gamma(n-|k|+2\gamma+1)}\frac{1}{2}\left[\left(\frac{kE}{\gamma\mu c^2}\right)^2 + \frac{kE}{\gamma\mu c^2}\right]}. \tag{2.18}$$

As we can see from the details above, where there still exist many left-outs, it is quite labourious, if not somewhat cumbersome, to obtain the solutions of the relativistic Dirac-hydrogen-like equation and wave functions in (2.5) in which the spherical harmonics constitute the main ingredient. This situation could lead to difficulties in calculations and simulations problems with these wave functions. It is no doubt that in many instances the spherical harmonics functions in (2.5) are regarded as rules, but not as calculation or simulation objects, let alone transformation simulations of these kind of waves into other type of waves, based on the above expressions.

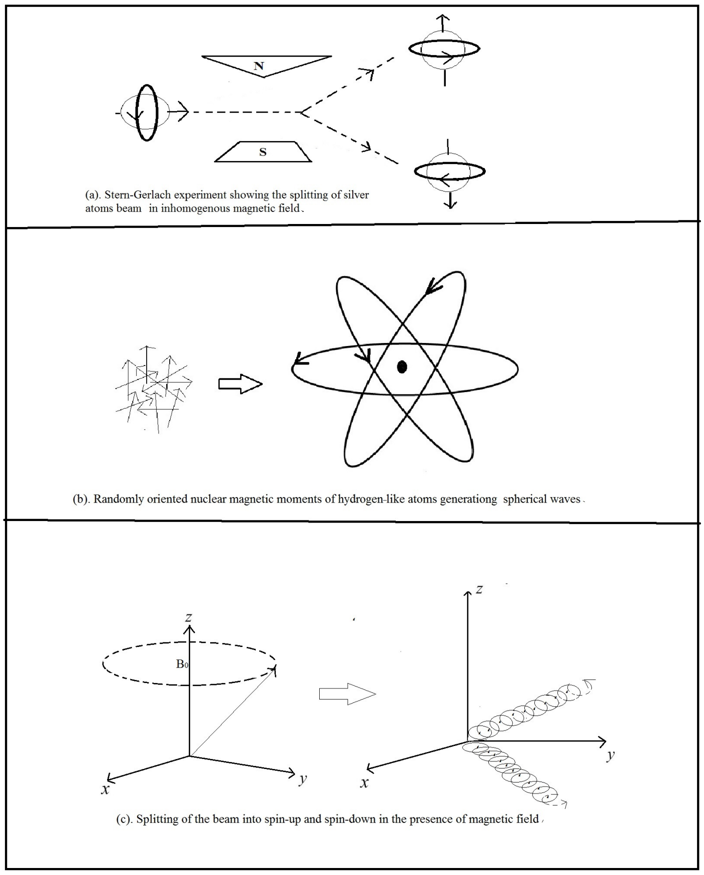

Fig. 1. Transformation from spherical waves into plane waves

3 STERN-GERLACH EXPERIMENT OR TRANSFORMATION FROM SPHERICAL WAVE EXPANSION INTO HELICAL PLANE WAVE FUNCTIONS

We leave the history and description of this famous experiment to the references [16], and lay our focus on its function, in order to uncover the analytical transformation underlying its phenomenon. A single beam of hydrogen-like (silver) atoms in ground state is guided through an applied transverse magnetic field and experiences, at the exit of this field, a splitting of its wave (which here is a spherical wave expansion) into two, representing spin-up and spin-down angular momenta, Fig.1.(a) and (c), as scientific observation and analysis made it clear and approved since 1922; and no other result has ever since contradicted this scientific accurate interpretation.

Firstly in the attempted interpretation , scientists generally admit that, at the exit from the magnetic field, the randomly oriented nuclear magnetic moments (in the initial beam, Fig.1.(b)) experience an external magnetic torque which tends to align the individual magnetic moment parallel or anti-parallel to the vertical $z-$direction of the applied field, hence the two optional orientations in spin-up and spin-down.

Secondly, it has been established that the phenomenon of spin and magnetic moment are non-intrinsic to the electron and do not depend on its internal structure. In particular, the spin angular momentum, proportional to and co-linear with the magnetic moment, is generated by a circulating flow of energy, forming a helical plane wave field (perpendicular to the direction of propagation) which carries the electron (or atom); likewise the magnetic moment is produced by a circulating flow of charge in the wave field of the electron [7] [8] (Section III).

Now, as the question is about magnetic torque leading to electron spin (i.e., mathematically, rotation resulting in spin), we are reduced to admit that the ground state atomic hydrogen-like spherical waves have undergone a process of *spin transformations* turning these waves into helical plane waves. Here, the term "spin transformations" is not a mere play on words. They are mathematical entities which have been derived in the context of determining the relationship which links the physical (realistic) process of three-dimensional spatial rotations to the "non-realistic" subatomic phenomenon of electron spin [12]; they are specifically rotation matrices of angle $2n\pi$, with $n = 1, 2, ...$

We now call in the Dirac-hydrogen helical plane wave function which has been obtained (in Paper (II), Eq. (3.35), of this series) as

$$\psi_{plane} = N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\sigma_z p_z}{mc^2+E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(p_z z - E_p t\right)/\hbar\right]. \tag{3.1}$$

Explicitly, in four dimensions [i.e., (1-time, 3-space), using a 4×4 Lorentz rotation matrix] together with the spherical waves in equation (2.5) which we now denote by $\psi_{spherical}$, and considering waves propagating in the $z-$direction with *positive energy*, we have

$$R_S \psi_{spherical} = \psi_{plane}, \tag{3.2}$$

where R_S is a 4×4 spin transformation matrix about the $z-$axis. That is,

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2n\pi) & -\sin(2n\pi) & 0 \\ 0 & \sin(2n\pi) & \cos(2n\pi) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} g_{n,k}(r)\, r^{-1} \sqrt{\frac{k+\frac{1}{2}-m}{2k+1}}\, Y_{k,m-1/2}(\theta,\phi) \\ -g_{n,k}(r)\, r^{-1} sgn\, k \sqrt{\frac{k+\frac{1}{2}+m}{2k+1}}\, Y_{k,m+1/2}(\theta,\phi) \\ if_{n,k}(r)\, r^{-1} \sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}}\, Y_{-k,m-1/2}(\theta,\phi) \\ -if_{n,k}(r)\, r^{-1} sgn\, k \sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}}\, Y_{-k,m+1/2}(\theta,\phi) \end{pmatrix}$$

$$= N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\sigma_z p_z}{mc^2+E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(p_z z - E_p t\right)/\hbar\right]. \tag{3.3}$$

Coming to equation (3.3) , the expression at the $r.h.s.$ includes only constant terms (except for the time variable t), not forgetting that the expression of the energy E_p involves the constant radius r; but the radial functions $g_{n,k}(r)$ and $f_{n,k}(r)$ are not constant. However, in the process of this spin transformation, it will take some time t for the spherical waves to be completely stabilized into helical plane waves; in this time interval, the radial functions become constant and identity functions of argument r, i.e., $g_{n,k}(r) = f_{n,k}(r) = r$ (the distance of the valence electron from the nucleus), showing the reason why t appears at the $r.h.s.$ of (3.3) but not at the $l.h.s.$ We deduce that equation

(3.3) is well-defined. Next, ∀n∈ $\mathbb{N}$, i.e., $n = 1, 2, ...$, we have that $\cos(2n\pi) = 1$ and $\sin(2n\pi) = 0$. All these observations put together reduce equation (3.3) to:

$$\begin{pmatrix} \sqrt{\frac{k+\frac{1}{2}-m}{2k+1}} Y_{k,m-1/2}(\theta,\phi) \\ \sqrt{\frac{k+\frac{1}{2}+m}{2k+1}} Y_{k,m+1/2}(\theta,\phi) \\ \sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}} Y_{-k,m-1/2}(\theta,\phi) \\ \sqrt{\frac{-k+\frac{1}{2}-m}{-2k+1}} Y_{-k,m+1/2}(\theta,\phi) \end{pmatrix} = N \begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \frac{c\sigma_z p_z}{mc^2+E_p} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \exp\left[i\left(p_z z - E_p t\right)/\hbar\right]. \quad (3.4)$$

Here, it should be observed that we have restricted the demonstration to the transformation from spherical waves into plane helical waves in the spin-up upward $z-$direction. For the case of spin-down, one uses the other unit component spinor $\chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ at the right hand side of (3.4).

Now, equation (3.4) holds iff the second and fourth entries in the 4-vector to the left are identically zero, since their corresponding counterpart to the $r.h.s.$ are. But since the spherical harmonic functions do not vanish, it follows that their coefficients (involving the quantum numbers) must be invalidated, leading to: $k + m = -1/2$ for the second entry and: $k + m = 1/2$ for the fourth, that is, in a combined form

$$|k + m| = \frac{1}{2}. \quad (3.5)$$

Assuming the quantum numbers are given and the terms involved in the expression of the plane wave function are known, we could expect equation (3.4) to have somewhat an equilibrium solution. It follows that the type of spherical wave functions which hold in the spin transformation from spherical wave expansion into helical plane wave functions in the SGE is the class of those with quantum numbers l, k, and m satisfying equation (3.5) ,and showing that spin transformation from spherical wave expansion into plane wave functions in the SGE is topical experimentally as well as analytically. However, at this stage, it seems somewhat cumbersome if not impossible, as mentioned above, to implement this observation in a simulation based on the spherical harmonics parameters, a task we will leave for future investigation.

4 CONCLUSION

One of the fundamental aspect to note about the Stern-Gerlach experiment, which has escaped scientific scrutiny so far, turns out to be the identification of its seminal spin transformation from ground state atomic spherical wave expansion into helical plane wave functions through a transversely applied magnetic field, and the analytical computations which underlie it. Of course, it is out of no pretentiousness to point this out if we understand one of its application could lead to a superluminal phenomenon involving a hydrogen-like particle, a phenomenon which is a measure of our net advance in science. For example, this could provide a ground for the understanding of the volatility of the hydrogen atom in many scientific experiments. One now only has to draw thereafter other essential scientific applications for own field of interest.

COMPETING INTERESTS

Author has declared that no competing interests exist.

References

[1] Einstein A. $\ddot{U}$ber einen die erzeugung und verwandlung des lichtes betreenden heuristischen gesichtspunkt. Annalen der Physik.1905;17:132148. Reprinted in (Stachel. Doc. 14, pp. 1989; 150169).

[2] Einstein A, Ehrenfest P. Quantentheoretische bemerkungen zum experiment von stern und gerlach. Zeitschrift fr Physik. 1922;11:3134. Reprinted in (Kormos Buchwald et al. 2012, Doc. 315).

[3] Heisenberg W. Zur quantentheorie der linienstruktur und der anoma- len Zeemaneekte. Zeitschrift fr Physik. 1922;8:273-297.

[4] Heisenberg W. $\ddot{U}$ber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitschrift fr Physik. 1927;43(3):172198.

[5] Phipps TE, Stern O. $\ddot{U}$ber die Einstellung der Richtungsquan- telung. Zeitschrift fr Physik. 1932;73:185-191.

[6] Belinfante F. On the covariant derivative of tensor-undors. Physica. 1940;7(4):305-324.

[7] Ohanian HC. What Is Spin?. New York; 1984.

[8] Gazoya EDK. Effect of cherenkov radiation on superluminal free spin-half particles motion in spacetime. Global Journal of Science Frontier Research: F. Mathematics and Decision Sciences. 2016;16(5).

[9] Dirac PAM. The quantum theory of the electron, Part 1. Proc. Roy. Soc. (London). 1928;A117:610-624. Part two. Proc. Roy. Soc. (London). 1928;A118:351-361.

[10] Greiner W. Relativistic quantum mechanics-wave equations (3rd ed.). Berlin, Heidelberg: Springer; 2000.

[11] Naber GL. The geometry of minkowski spacetime. New York: Springer-Verlag; 1992.

[12] Gazoya EDK, Prempeh E, Banini GK. The one-parameter subgroup of rotations generated by spin transformations in three-dimensional real space. Journal of Ghana Science Association. 2015;16(1).

[13] Gerald T. Mathematical methods in quantum mechanics; With applications to Schrdinger operators. American Mathematical Society; 2009.
ISBN 978-0-8218-4660-5

[14] Tipler P, Ralph L. Modern physics (4th ed.). New York: W. H. Freeman and Company; 2003.
ISBN 0-7167-4345-0

[15] Courant R, Hilbert D. Methods of mathematical physics. Wiley-Interscience. 1962;I.

[16] Horst SB, Lothar S, Hans JL, Wolfgang T, Alan T, Tilman S. The Stern-Gerlach experiment revisited. 2016; Xiv: 1609.09311v1 [physics.hist].

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