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Study of $f(T)$ Mass Function of Nearby Galaxy Clusters: Determining σ_8

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This paper presents a new analytical method to determine amplitude of density fluctuations of 152 nearby clusters ($z \leq 0.15$). We investigate the rms linear fluctuation in the mass distribution on scales of $8h^{-1}Mpc$ i.e. σ_8 , by using Press-Shechter mass function. The mass function is estimated for masses larger than $M_{lim} = 4 \times 10^{14} h^{-1} M_{\odot}$. We find rms density fluctuation equal to 0.52 for the critical density universe. The results found are consistent with those, obtained with alternative models for the high density universe. The results agree with the previous papers obtained from different models and considerations. This takes us to introduce a new approach to estimate the cosmological parameters. For critical density , the slight variation in results may be due to the fact that there are observational uncertainties in estimates of cluster masses, which are in general not neglegible.

Keywords: Galaxies; clusters; general - cosmological parameters theory - large-scale structure of the universe.

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1 INTRODUCTION

The largest virialized structures at present are the clusters of galaxies. One of the most fundamental predictions of hierarchical structure formation is the mass function: the number density of objects as a function of their mass M. The abundance of clusters as a function of mass and redshift has been shown to be a sensitive probe of cosmological parameters like density parameter (ω_m) and rms density fluctuaton σ_8 [1, 2, 3, 4, 5]. In order to link the observed mass function of clusters to an underlying cosmology, one must either use an analytic description of cluster abundance or many numerical studies [6, 7, 8]. Although one of the most commonly used descriptions of cluster halo abundance is the Tinker et.al. (2012), we shall appeal to the description of Press and Shecter [9].

Galaxy clusters can be effeciently used as tools for estimating fundamental cosmological parameters because of their relative dynamical youth. The mass within the virial radius of a rich cluster $M_{lim} = 5 \times 10^{14} h^{-1} M_{\odot}$ is very close to the mass enclosed within a sphere of radius $8h^{-1}Mpc$. Because of this the present day abundance of the rich clusters directly reflects the amplitude of density fluctuations on a scale of $8h^{-1}Mpc$ and can be used to measure this amplitude with a minimum of assumptions. It does however depend on the value of the cosmological density parameter Ω_0. Thus the local cluster abundance fixes the value of σ_8, the rms density fluctuation in spheres of radius $8h^{-1}Mpc$, as a function of Ω_0 [10].

In this work we aim to investigate the constraints on the power spectrum of the cosmic density fluctuations, by estimating the quantity related to the power spectrum i.e. dispersion of the density field σ_R at $R = 8h^{-1}Mpc$. This analysis is based on the virial mass estimated by [11], (hereafter G98), for a sample of 152 clusters. In section 2 we discuss the cosmological parameter σ_8 . In section 3 we apply the Press-Shecter mass function to these clusters and estimate the value of σ_8. In section 4 we discuss the results and finally in section 5 we present the conclusion. A Hubble constant $H_0 = 100$ h kms^{-1} Mpc^{-1} is used in this work.

2 POWER SPECTRUM

There have been rapid advances in observational cosmology, leading to a precison cosmological model and many cosmological parameters determined to one or two significant figure accuracy. In this determination, the observational cosmologist utilizes the astronomical information to derive cosmological parameters. This method of transformation from the observables to the parameters, usually involves many assumptions about the nature of the data. A successful cosmological model of the universe should include a description of deviations from homogeneity, at least in a statistical way. Indeed, some of the most powerful probes of the parameters study the evolution of perturbations, as it is intertwined with the determination of cosmological parameters. Other than the large-scale primordial amplitude, there are different ways in which the density perturbation amplitude can be specified. For example studying its effect on the Cosmic Microwave Background (CMB) or by specifying a short-scale quantity, a common choice being the present linear-theory mass dispersion on a scale of $8h^{-1}Mpc$, known as σ_8. In the heirarchical clustering, once the perturbations have developed to amplitude greater than some critical value δ_c, they develop rapidly into bound objects with mass M. The perturbations can then be assumed to have a power law spectrum, growing with time up to the present epoch. The rms linear density fluctuation σ_M is related to the fluctuation power spectrum $P(k)$ as

$$\sigma_M^2 = (2\pi^2)^{-1} \int_0^{\infty} \mathrm{d}k k^2 P(k) W^2(kR) \quad (2.1)$$

where k is the wave number and R is the comoving fluctuation size. Here $W(kR)$ is the Fourier transform of the window function, which describes the shape of the volume from which the collapsing object is accreting matter. Finally $W(kR)$ is assumed to have the top hat profile, given by

$$W(kR) = \frac{3(\sin kR - kR\cos kR)}{(kR)^3} \quad (2.2)$$

Accordingly the mass M of a cluster arising from the collapse of a fluctuation with typical size R is

$$M = \frac{4\pi}{3}\rho_{c0}R^3 \quad (2.3)$$

where ρ_{c0} is the average matter density.

The dependence of the power spectrum on the wavenumber k is usually written as

$$P(k) = AkT^2(k) \quad (2.4)$$

where $T(k)$ is the transfer function and A is a normalization constant. The transfer function depends both on the cosmological parameters as well as on the cosmic matter constituents e.g. fraction of cold, hot and baryonic matter, number of relativistic species. For a pure Cold Dark Matter (CDM) model, $T(k)$ depends to a good approximation only on the shape parameter $\Gamma = \Omega_0 h$ (e.g. Bardeen et al. 1986), for neglegible baryon concentration. The amplitude of $P(k)$ is usually expressed in terms of σ_8, the r.m.s. density fluctuation within a top-hat sphere of $8h^{-1}Mpc$ radius. The measured rms fluctuation in galaxy numbers within a sphere of radius $8h^{-1}Mpc$ reaches close to unity. Because of this the rms linear fluctuation in mass σ_8 is taken as a measure of amplitude of density fluctuations.

The cosmological parameters are helpful tools that help us to track the evolving history of the universe, back to an epoch where interchanges between the densities of the different species are allowed by interactions. This is believed to have happened shortly before Big-Bang Nucleosynthesis (BBN) probably at neutrino decoupling [12]. In the present paper we are discussing the accuracy of power spectrum amplitude σ_8 analytically, by using Press-Shecter Mass Function.

3 PRESS-SHECTER MASS FUNCTION

The Press-Shecter technique is an excellent analytic approach to describe the evolution of the cluster mass function, which has been extensively used against N-body simulations and found to fare extremely well [13]. The theory of Press and Shecter (1974) describes the evolution of the cluster mass function n(M) of cosmic structures, defined by

$$dN = n(M)dM \quad (3.1)$$

where dN is the number of structures per unit volume with mass between M and $M + dM$. The Press and Shecter analysis begins with the assumption that, when the purterbations have developed to amplitude greater than some critical value δ_c, they rapidly develop into bound objects with mass M. The perturbations have a power-law power-spectrum $P(K) = k^n$ which follow the rules that describe the growth of the perturbations with cosmic epoch. Press and Shecter assumed that the primordial density fluctuations are Gaussian fluctuations and are described by Gaussian probability distribution:

$$P(\delta_M)d\delta_M = \frac{1}{(\sqrt{2\pi})\sigma_M}\exp\left(-\frac{\delta_M^2}{2\sigma_M^2}\right)d\delta_M \quad (3.2)$$

where $\delta = \frac{\delta_\rho}{\rho}$ is the density contrast associated with a perturbation of mass M. The probability that at some future time δ_M exceeds some critical value δ_c is now given by

$$P_{>\delta_c}(M) = \int_{\delta_c}^{\infty} P(\delta_M)d\delta_M \quad (3.3)$$

The probability $P_{>\delta_c}$ is proportional to the number of cosmic structures with density perturbations $> \delta_c$. A value of $> \delta_c = 1.68$ is of great importance because this is the value of density pertubations which corresponds to virialized structures. To find the number of structures with mass M , which are isolated i.e. surrounded by underdense regions, we must subtract the term $P_{>\delta_c}(M + dM)$. The underdense regions i.e. those with $\delta < \delta_c$, are not properly accounted for, which means only half of the total mass density being condensed into bound objects. Press and Shecter solved this problem by multipling the mass function by a factor of 2. As mentioned above, we can now develop the expression for mass function $n(M)$, by the difference of the probabilities:

$$n(M)dM = 2[P_{>\delta_c}(M) - P_{>\delta_c}(M + dM)] \quad (3.4)$$

To convert probabilities to units of per volume, we multiply the above equation by ρ_{c0}/M

$$n(M)dM = \quad 2\frac{\rho_{c0}}{M}[P_{>\delta_c}(M) - P_{>\delta_c}(M+dM)] \tag{3.5}$$

The above equation can be written as

$$n(M)dM = -2\frac{\rho_{c0}}{M}\frac{dP_{>\delta_c}}{dM}dM \tag{3.6}$$

where ρ_{c0} is the present day density of the matter.

Equation (3.6) can further be written as

$$n(M)dM = \quad -2\frac{\rho_{c0}}{M}\frac{dP_{>\delta_c}}{d\sigma_M}\frac{d\sigma_M}{dM}dM \tag{3.7}$$

Using equation (3.3), we evaluate the derivative $\frac{dP_{>\delta_c}}{d\sigma_M}$ as

$$\frac{dP_{>\delta_c}}{d\sigma_M} = \quad \frac{d}{d\sigma_M}\int_{\delta_c/\sqrt{2}\sigma_M}^{\infty} P\delta_M d\delta_M \tag{3.8}$$

Again using equation (3.2), we proceed to obtain the final result of the derivative $\frac{dP_{>\delta_c}}{d\sigma_M}$

$$\frac{dP_{>\delta_c}}{d\sigma_M} = \quad \frac{1}{\sqrt{2\pi}}\exp(-\frac{\delta_c^2}{2\sigma_M^2})\frac{\delta_c}{\sigma_M^2} \tag{3.9}$$

where we have used the fundamental theorem

$$d/dx\int_a^x f(t)dt = f(x) = -d/dx\int_x^a f(t)dt \tag{3.10}$$

Substituting equation (3.9) into equation (3.7) yields the required mass function

$$n(M)dM = -\sqrt{\frac{2}{\pi}}\frac{d\sigma_M}{dM}\frac{\rho_{c0}}{M}\frac{\delta_c}{\sigma_M^2}\exp(\frac{\delta_c^2}{2\sigma_M^2})dM \tag{3.11}$$

where $n(M)$ is the number density of collapsed objects per unit mass, σ_M is the variance of the density field or root mean squared fluctuation, filtered on a scale R enclosing a mass M at redshift z and ρ_{c0} is the critical density of the universe at the present epoch. A value of δ_c of great interest to us is $\delta_c = 1.686$ since we have seen that this is the critical linear-theory density contrast needed for collapse, which corresponds to virialized structures. The normal procedure is to evaluate n(M) by calculating σ_M and its derivative from the linear theory matter power spectrum.

Determining $n(M)$ from galaxy clusters has been an important problem in cosmology. Different techniques are avaiable for determining $n(M)$. One method is to find the normalization of the matter power spectrum through the parameter σ_8, that is , σ_R evaluated for $R = 8h^{-1}Mpc$ at z=0. For determining n(M) the next thing we need is the derivative of σ_M . It is simplest to use an approximation to the true shape of $\sigma_M = \sigma_R$ in the vicinity of $R = 8h^{-1}Mpc$. Particularly for low densities , we require accuracy over a greater range of scales, we use the more accurate fit, with the scalar spectral index $n_s = 1$ as [14]:

$$\sigma_R = \sigma_8(\frac{R}{8h^{-1}Mpc})^{-\gamma(R)} \tag{3.12}$$

where

$$\gamma(R) = (0.3\Gamma + 0.2)[2.92 + \log(\frac{R}{8h^{-1}Mpc})] \tag{3.13}$$

with $\Gamma = 0.230$

Now the mass given by equation (2.3), corresponds to a sphere of radius,

$$R = [\frac{3M}{4\pi\rho_{c0}}]^{1/3} \tag{3.14}$$

Where ρ_{c0} is the critical density at the present epoch given by $\rho_{c0} = 2.78 \times 10^{11} h^2 M_\odot Mpc^{-3}$.

For evaluating the mass function n(M), given by equation (3.11) we need to evaluate the derivative $\frac{d\sigma_M}{dM}$. We proceed as:

$$\frac{d\sigma_M}{dM} = \frac{d\sigma_R}{dR}\frac{dR}{dM} \tag{3.15}$$

Now using equation (3.12) in equation (3.15), we obtain

$$\frac{d\sigma_M}{dM} = -0.77\frac{\sigma_R}{R}(\frac{dR}{dM}) \tag{3.16}$$

Again using equation (3.14) in the above equation, we obtain the required derivative $\frac{d\sigma_M}{dM}$:

$$\frac{d\sigma_M}{dM} = -\frac{0.77}{3}\frac{\sigma_R}{M} \tag{3.17}$$

Inserting equation (3.17) into equation (3.11) yields the desired mass function

$$n(M)dM = \sqrt{\frac{2}{\pi}}\frac{\rho_{c0}}{M^2}\frac{0.77}{3}\frac{\delta_c}{\sigma_R}\exp(\frac{-\delta_c^2}{2\sigma_R^2})dM \tag{3.18}$$

This is a very useful formalism for studying the development of galaxies and clusters of galaxies in hierarchical scenarios for galaxy formation. Now we compare this function with the observed cluster abundance of the 152 clusters of G98. To equate it with the observed number density of these clusters with units of per mass, we need to divide the number density by the mass scale M . The resulting number density per unit mass comes out to be equal to

$$n(M)dM = 6.3 \times 10^{-6} M^{-1} h^3 Mpc^{-3} \tag{3.19}$$

Let us now equate this cluster number density with the number density as given by Press-shecter mass function of equation (3.18). Proceeding in this direction the result is

$$x\exp(\frac{-x^2}{2}) = 0.04 \tag{3.20}$$

with $x = \frac{\delta_c}{\sigma_R}$.

This equation can be solved numerically. A simple way of doing this is to write it as

$$x = \sqrt{2\ln(\frac{x}{0.04})} \tag{3.21}$$

We now start iterations. Making a first guess for x we plug it on the right hand side . Then we use the output as the input for the next iteration. When the result changes little from one iteration to the next, a solution has been found. Using the repeated iterations we found the solution $x = 2.9$.

Also from equation (3.14), by substituting the value of cluster mass, $M_{lim} = 4 \times 10^{14} h^{-1} M_\odot$ we get $R = 7.00$.
So

$$x = \frac{\delta_c}{\sigma_R} = 2.9 \tag{3.22}$$

Since $\delta_c = 1.686$, we obtain

$$\sigma_R = \frac{1.686}{2.9} = 0.58 \tag{3.23}$$

Finally we can now use equation (3.12) to find the rms density fluctuation σ_8. Sustituting equation (3.23) into equation (3.12) yields

$$\sigma_R = 0.58 = \sigma_8(\frac{R}{8})^{-0.77} = \sigma_8(\frac{7.00}{8})^{-0.77} \tag{3.24}$$

and we calculte the result as $\sigma_8 = 0.52$

This is the value of the amplitude of density fluctuations for the given sample of 152 clusters. The resuts we have obtained are consistent with those obtained by other authors from the cluster abundance, with alternative models, for the high density universe. Eke et.al.(1996) and Markevitch (1998) found a similar normalization of $\sigma_8 = 0.52 \pm 0.04$ for $\omega_0 = 1$, based on the analysis of the cluster temperatures by Henry and Arnaud (1991) [15, 16, 17]. A slightly higher normalization $\sigma_8 = 0.57 \pm 0.05$ for $\omega_0 = 1$ has been obtained by White et.al. (1993) from the median velocity dispersion of Abell clusters, as provided by Girardi et.al. (1998) and from the temperature functions of Henry et.al. (1991) and Edge et.al. (1990) [18, 19]. Oukbir et.al. (1997) found the same result as White et.al. (1993) [20]. Almost similar results $\sigma_8 = 0.58 \pm 0.02$ for $\omega_0 = 1$ were obtained by Borgani et.al. (1999) [21]. But considerably higher results $\sigma_8 = 0.748 \pm 0.035$ have been found by Hudson et.al. (2012), by using peculiar velocities as a probe of the growth rate of mass density fluctuations in the universe [22]. Still higher results have been obtained by Planck Collaboration 2018; I and VI: $\sigma_8 = 0.811 \pm 0.006$ [23, 24]. It is remarkable that all these results, except Hudson et.al. (2012) and Planck Collaboration 2018 results, although based on largely different data sets, give almost the same normalization.

4 DISCUSSION

The masses of rich clusters of galaxies provide a sensitive measurement of the amplitude of linear fluctuations in mass on a standard scale of $8h^{-1}Mpc$, where galaxy fluctuations have near unit amplitude. This measurement is almost independent of the shape of the fluctuation spectrum and depends primarily on the Gaussion nature of the initial density field and on the mean cosmological density. But in order to do this one must be able to determine the mass of clusters accurately. Since the dominant contribution to the mass of a cluster is in the form of dark matter, this seems to be a non-trivial task.

We have presented a new analytical determination of the mass function n(M) of nearby galaxy clusters. We have applied the Press-Shecter formalism to 152 nearby clusters. This analysis is based on the mass estimates by Girardi et.al. (1998) for a sample of 152 clusters. Applying Press-Shechter theory, we determine the number density of the given sample of 152 clusters. Then we compare it with the observed number density of these clusters as provided by Girardi et.al. (1998). By comparing the two number densities, we determine the amplitude of the fluctuation power spectrum σ_8 and find out that $\sigma_8 = 0.52$. The results are consistent with those, obtained with alternative models for the high density universe . Our results closely match with those from Eke et.al. (1996), White et.el. (1993), Edge et.al. (1990), Oukbir et.al. (1997) and Borgani et.al. (1999). But our results are lower than Hudson et.al. (2012) and Planck Collaboration 2018 Results I and VI.

However the precise value of σ_8 is still to be determined because there are several uncertainties in the values of cluster number density and the masses of clusters which propagate to an uncertainty in σ_8. There are several errors in this estimate, which are difficult to quantify , since they are entirely due to systematic uncertainties in the mass estimates for rich clusters of galaxies. For example to determine the number density of galaxy clusters, we find the normalization of the matter power spectrum through the cosmological parameter σ_8 i.e. σ_R evaluated at $R = 8h^{-1}Mpc$. But for doing that we should determine the mass of the galaxy clusters. Since the dominant contribution to the mass of a cluster is in the form of dark matter, so this is a difficult task. Again there are uncertainties in the values of number density and mass of clusters which propagate to an uncertainty in the determination of σ_8. A reason of caution when comparing data and model predictions is related to the observational uncertainties in estimates of cluster masses, which are in general not neglegible.

In future we intend to increase the sample of clusters, by taking 800 nearby clusters from Sloan Digital Sky Survey (SDSS) for the investigations. We intend to determine their dynamical parameters like, mass, virial radius , number density and other parameters, eventually leading us to determine the normalization σ_8. We also intend to use other mass functions like Sheth and Tormen (2002) and Tinker et.al. (2008) to determine the cosmological parameters [25]. The problem is a challenging one and will throw light on the large scale structure of the universe in a more profound way.

5 CONCLUSION

Cosmological parameters are forever increasing in the scope, and nowadays include the parameterization of some functions, as well as simple numbers describing properties of the Universe. The original usage was limited to describing the global dynamics of the Universe, such as its expansion rate and curvature. They help us to know how the matter budget of the Universe is built up from its constituents: baryons, photons, neutrinos, dark matter, and dark energy. The parameters also lead to describe the nature of perturbations in the Universe, through global statistical descriptors such as the matter and radiation power spectra. Typical comparisons of cosmological models with observational data now feature between five and ten parameters.

In this work we estimate the cosmological parameter σ_8, because in the linear theory it is one of the fundamental parameters to describe the power spectrum of mass fluctuations in the universe. In the large scale structure simulations, as well, it is one of the key parameters.

In the present paper we determine the cosmological parameter σ_8 by applying Press-Shecter mass function to a sample of 152 clusters. We find out that $\sigma_8 = 0.52$. It is remarkable that the estimated results closely match with those obtained with alternative models such as galaxy-galaxy correlations, fluctuations in the CMB, gravitational lensing statistics and galaxy peculiar velocities. However our results are lower than Planck Collaboration 2018 results. This may be due to the fact that there are several uncertainties in the masses and number densities of clusters which affect the precise value of σ_8. In future this can be taken care of by taking a larger sample of clusters.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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