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The Adjunct Force of Gravity

John A. T. Bye[1*]

[1]The University of Melbourne, Victoria, 3010, Australia.

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Short Research Article

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ABSTRACT

In the paper, we show that there are two types of gravitational force. The axial Newton force of gravity, focussed on the primary mass, and also the circumferential Prandtl adjunct force of gravity, which derives from the assembly of masses other than the primary mass.

The Newton force occurs in a stress-free planetary environment, and the Prandtl force, which is proportional to the square of van Karman's constant (κ) is a result of the inter-planetary stresses which occur due to the formation of the planets. In the planetary system the Prandtl force is much smaller in magnitude than the Newton force, but in the universe it is much greater than the Newton force due to any primary mass.

The Newton force gives us, not only the law of gravity, but also the basic structure of the planetary system, for which, we show that the product of the annular density, $\rho(R)$, and the radius (R), $R\,\rho(R)$ is a constant, as is demonstrated by the orbital properties of the planets.

On the other hand, the Prandtl force is much greater than the Newton force in the Einstein model for the universe due to its infinitely greater mass in comparison with any Newton primary mass. This axiom applied in Newton's theory of gravity leads directly to Einstein's famous law, $E = M\,c^2$, where E is energy, M is mass, and c is the speed of light, which is a constant, and also to a companion law for torque, $Q = \kappa^2/8\,\pi M c^2$, where $Q = RJ$ in which J is the standard deviation of the circumferential Prandtl force.

In the numerical evaluations, we use the consensus experimental value ($\kappa = 0.4$) throughout.

**Corresponding author: E-mail: jbye@unimelb.edu.au;*

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1. INTRODUCTION

We live in an environment in which there are two states. In State (A) there is mass, but no shear stress; and in State (B) there is no mass, but there is shear stress.

In this paper we develop the physics of this environment in which State (A) will be referred to as the Newton model, and State (B) will be referred to as the Prandtl model. For both states, the nomenclature follows that of the founders who proposed the physics which occurs.

The physics is based on two principles: (i) the conservation of mass, which gives rise to the Newton force of gravitation, and (ii) the inertial coupling relation for the interaction of two fluids of large density contrast (I), which gives rise to the Prandtl shear force, which we will call the adjunct gravitational force.

The analysis shows how these two principles are of equal importance. This is the first use of principle (ii) in cosmology, although it has long been used in oceanography and meteorology, where the two contrasting fluids are air and water, as summarized in Bye [1].

Section 2 shows how the inertial coupling relation leads to the Prandtl model for the friction velocity, and Section 3 presents the physics in a cylindrical co-ordinate frame suitable for cosmological investigations. A fundamental step in the analysis is the averaging of the mass of a planet around the ring in which it orbits. This procedure is analogous to that in hydrodynamics for fluids of uniform density, where the shear stress is determined by the normal gradient of velocity.

2. INERTIAL COUPLING AND PRANDTL'S MIXING LENGTH THEORY

The inertial coupling relation for the coupling of two fluids of large density contrast [2] is,

$$u_* = K_I^{1/2}\left(\frac{(\rho_1^{1/2}\,(u_1 - u_o) - \rho_2^{1/2}(u_2 - u_o))}{\rho_1^{1/2}}\right) \qquad \text{(I)}$$

in which u_1 and u_2 are respectively the surface wind and the surface current velocities, u_o is a reference velocity, ρ_1 and ρ_2 are respectively the densities of air and of water, and K_I is the inertial drag coefficient. The inertial coupling relation (I) represents the interaction of two fluids of unequal density, such as air and water in aerodynamically rough flow in which the effects of viscosity can be neglected.

In a continuum, for a boundary layer, $-z_B \leq z \leq z_B$, centred on $z = 0$ in which $\rho(z_B) = \rho_1$ and $\rho(-z_B) = \rho_2$, and $\rho_1^{1/2}/\rho_2^{1/2} = 1 - \Delta\rho/\rho$, where $\Delta\rho = \rho_2 - \rho_1$ and $\rho = ½(\rho_1 + \rho_2)$, we have to O(1) from (I),

$$u_* = K_I^{1/2}\left((u_1 - u_o) - \left(1 + \frac{1}{2}\frac{\Delta\rho}{\rho}\right)(u_2 - u_o)\right)$$

Bye and Wolff [3] from which, on defining the mean velocity, $u = ½(u_1 + u_2)$, and substituting $(u_1 - u_o) = (u - u_o + ½\,\Delta u)$ and $(u_2 - u_o) = (u - u_o - ½\,\Delta u)$ we obtain the differential relation, $u_* = K_I^{1/2}(du + ½\,u/\rho\, d\rho)$ where $du = \Delta u$ and $d\rho = -½\Delta\rho$ which can be expressed as

$$u_* = 2\, z_B \left(\frac{K_I}{\rho}\right)^{1/2} \left(\frac{d(u - u_o)\rho^{1/2}}{dz}\right)_{z=z_B}$$

On assuming that this relation is valid for $z \geq z_B$ and that $u_o = 0$, we obtain the Prandtl model for the friction velocity,

$$u_* = \frac{L}{\rho^{1/2}}\frac{d(u\rho^{1/2})}{dz} \qquad \text{(II)}$$

where $L = \kappa z$ is the mixing length, and κ is von Karman's constant, from which the shear stress,

$$\tau_{zx} = L^2 \left|\frac{d(u\rho^{1/2})}{dz}\right| \frac{d(u\rho^{1/2})}{dz}$$

which for a constant density (ρ) is known as Prandtl's mixing length hypothesis [4] as it results from the interaction of large agglomerations of fluid particles within the fluid. The Prandtl mixing length hypothesis is used universally in boundary layer studies in the atmosphere and the ocean, see for example. Garratt [5] and Jones and Toba [6], and a full account of the development of the inertial coupling theory for momentum exchange between the ocean and the atmosphere is presented in Bye [1].

In this cosmological study we apply the Prandtl model in its full extent in which the stress is produced by the interaction of large agglomerations of particles whose density varies significantly, in contrast to within the atmosphere and the ocean, where the variation of density in (II) is negligible.

3. APPLICATION OF THE PRANDTL MODEL IN A CYLINDRICAL CO-ORDINATE FRAME

3.1 The Inertial Coupling Relations

In a cylindrical co-ordinate system (R, ϕ) where R is the radial co-ordinate and ϕ is the azimuthal co-ordinate, the Prandtl model for the friction velocity (II) in a continuum is,

$$u_* = \left(\frac{L}{\rho^{1/2}} \frac{d(U\,\rho^{1/2}\,2\pi RW)}{dR}\right)(2\pi RW)^{-1} \qquad (1)$$

where U is the azimuthal velocity, ρ is the annular mean density, W is the thickness of the rotating disk, and $L = \kappa R$ is the mixing length. On substituting for L, and expanding the derivative, we obtain,

$$u_* = \frac{\kappa}{W}\left(\frac{d(UWR)}{dR} + \frac{UWR}{\rho^{1/2}}\frac{d\rho^{1/2}}{dR}\right) \qquad (2)$$

3.2.The Stress Free State

The mass conservation relation is,

$$dM = m\,dR \qquad (3)$$

in which *m* = *ρ 2πR W* and,

$$\frac{dm}{dR} = 0 \qquad (4)$$

guarantees mass conservation, and from which,

$$\frac{1}{\rho}\frac{d\rho}{dR} = -\frac{1}{RW}\frac{d(RW)}{dR} \qquad (5)$$

which yields,

$$\frac{1}{\rho^{\frac{1}{2}}}\frac{d\rho^{\frac{1}{2}}}{dR} = -½\,\frac{1}{RW}\frac{d(RW)}{dR} \qquad (6)$$

and on substituting (6) in (2) we have,

$$u_* = \frac{\kappa}{W}\left(\frac{d(UWR)}{dR} - ½\,U\frac{d(RW)}{dR}\right) \qquad (7)$$

3.3 Newton's Law of Gravitation

In a circular orbit, which has no spin at the origin $(R = 0)$, $U(R) = 2\pi R/T$ where T is the orbital period, and

$$\frac{R^3}{T^2} = A^2 \qquad (8)$$

where $A^2 = G(M_o + M)/4\pi^2$ in which M_o is the mass of the primary body, M is the external mass, and G is the universal gravitational constant. On substituting for U in (7) using the positive root of (8), we obtain,

$$u_* = \frac{\kappa}{W}\left(½R^{1/2}\frac{dW}{dR}\right)2\pi A \qquad (9)$$

In the stress-free state, (9) reduces to,

$$\frac{R^{\frac{1}{2}}}{W}\frac{dW}{dR} = 0 \qquad (10)$$

There are two solutions to (10): (i) the interior stress-free solution,

$$W = \text{constant} \qquad (11)$$

and (ii) the exterior stress-free solution, in which using the mass conservation equation (4),

$\frac{1}{W}\frac{dW}{dR} = -\frac{1}{(\rho R)}\frac{d(\rho R)}{dR}$. Hence, on substituting for A, (9) may be expressed as,

$$\rho^{1/2}\,u_* = -\kappa(G(M_o + M))^{1/2}\frac{d\,(R\rho)^{1/2}}{dR} \qquad (12)$$

where, $\tau_{R\phi} = \rho\,|u_*|\,u_*$ is the azimuthal shearing stress, and in the stress-free state $(\tau_{R\phi} = 0)$,

$$\frac{d\,(R\rho)^{1/2}}{dR} = 0 \qquad (13)$$

and the exterior stress-free solution is,

$$(R\rho)^{1/2} = \text{constant} \qquad (14)$$

Eq. (14) may also be expressed in terms of U using (8), which yields,

$$\frac{U^2}{\rho} = \text{constant} \qquad (15)$$

There are two limiting gravitational situations: (a) $M \gg M_o$, and (b) $M_o \gg M$ which are discussed below. From (4), mass conservation requires that

$$M = m\,R \tag{16}$$

Where m is a constant in each situation.

(a) The Einstein model ($M \gg M_o$)

For $M \gg M_o$, on substituting (8) in (15),

$$\frac{U^2}{\rho} = \frac{GM}{\rho R} \tag{17}$$

which, on using (16) yields,

$$U^2(R) = Gm \tag{18}$$

where $m = \rho 2\pi W R$. Eq. (18) is a universal relation as it applies anywhere in the disk domain. We identify $U(R)$ with the speed of light (c), from which it follows that,

$$m = \frac{c^2}{G} \tag{19}$$

which may be re-expressed, using (16) as,

$$P = Mc^2 \tag{20}$$

where $P = gMR$ is the potential energy of the rotating disk, in which $g = GM/R^2$ is the acceleration of gravity. Eq. (20) is precisely Einstein's famous law.

Eq. (15) shows, since $U(R)$ is a constant, that the density, $\rho(R)$ is also a constant, and hence since m is a constant, as $R \to \infty, W \to 0$, i.e. the Einstein universe is finite, and from (19) for $G = 6.673\ 10^{-11}$ m^3 kg^{-1} s^{-2} and $c = 2.998\ 10^8$ ms^{-1}, $m = 1.35\ 10^{27}$ kg m^{-1}.

The Einstein model is discussed further in the Appendix after the discrete Prandtl model has been introduced in Section (c).

(b) The Newton model ($M \ll M_o$)

In the Newton model, $M \ll M_o$ and hence (12) reduces to,

$$\rho^{1/2} u_* = -\kappa\,(GM_o)^{1/2} \frac{d\,(R\rho)^{1/2}}{dR} \tag{21}$$

which may be re-expressed in the alternate form,

$$\rho^{1/2} u_* = -½\,\kappa\,\frac{(GM_o)^{1/2}}{(R\rho)^{1/2}} \frac{d(R\rho)}{dR} \tag{22}$$

(c) The discrete Prandtl model

We interpret the variability in ρR as follows, by replacing the rotating disk of the base model with a series of rotating rings, each of which contains the mass of a planetary body, and a series of intervening rings, which are mass-free and support the shear stresses arising from the planetary model (22). Thus, there are two states, which are either: (A) 'massy' and stress-free, or (B) 'stressy' and mass-free. We turn our attention now to State (B).

In the continuum system, (13) is satisfied by a stress-free environment. In a discrete system, in which mass and stress have separated, (21) or (22) yield the two-body expression for u_*,

$$\rho^{1/2}\,u_* = -½\,\kappa\,\frac{(GM_o)^{1/2}}{(R\rho)^{1/2}} \frac{\Delta\,(R\rho)}{\Delta R} \tag{23}$$

where $(R\rho) = (R\rho)_1 - (R\rho)_2$, and $\Delta R = R_1 - R_2$ in which the suffices 1 an 2 refer to the two bodies, from which

$$\tau_{R\phi} = -¼\,\kappa^2 (GM_o) \left|\frac{\Delta\,(R\rho)}{\Delta R}\right| \left(\frac{\Delta\,(R\rho)}{\Delta R}\right) (\rho R)^{-1} \tag{24}$$

which will be applied to selected pairs of planets in the Planetary System in Section 3.4.

Over the complete planetary system, $\Delta R\rho = 0$, and hence, $\rho^{1/2}\,u_* = 0$ and $\tau_{R\phi} = 0$. In effect, the Prandtl force is a perturbation, normal to the stress-free Newton force, which arises in the discrete system.

In terms of forces, we have the Newton force of gravity on mass (M), directed towards the centre of the Sun,

$$F = -\frac{GM_o\,M}{R^2} \tag{25}$$

and the differential Prandtl force directed circumferentially between the two orbiting bodies,

$$dJ = \tau_{R\phi}\,W\,dR \tag{26}$$

Table 1. Orbital parameters for the Planetary System from Jones [8]

	M_P x 10^{23} kg	R x10^{11} m	ρ kg m^{-3}	D x 10^6 m	ρ R x 10^{11} kg m^{-2}	m x 10^{17} kg m^{-1}
Mercury	3.3	0.579	0.0485	4.88	0.0281	0.86
Venus	48.7	1.082	0.0623	12.1	0.0674	5.12
Earth	59.8	1.496	0.0498	12.76	0.0745	5.97
Mars	6.44	2.28	0.0124	6.8	0.0282	1.2
Jupiter	19000	7.783	0.0242	142.8	0.188	168.7
Saturn	5690	14.27	0.00561	120	0.08	60.3
Uranus	876	28.69	0.00231	51.8	0.0662	21.5
Neptune	1030	44.97	0.00192	49.2	0.0863	26.7

Hence, on differentiating (25) using the mass continuity relation, $\frac{dM}{dR} = 2\pi R\rho W$, and substituting for $\tau_{R\phi}$ from (24) in (26), we obtain,

$$\frac{dJ}{dF} = \frac{\kappa^2}{8\pi\rho^2}\left|\frac{\Delta(R\rho)}{\Delta R}\right|\frac{\Delta(R\rho)}{\Delta R} \qquad (27)$$

in which $K = \frac{dJ}{dF}$ is the ratio of the Prandtl force (dJ), which in generic terms, we call the two-body Newton adjunct force of gravitation, to the one-body Newton force of gravitation (dF).

In the continuum system, allowance for the mass of the rotating disk may be made by substituting $A = (G[M_o + M])^{1/2}/2\pi$ in (21) instead of $A = (GM_o)^{1/2}/\, 2\pi$, which yields,

$$\rho^{1/2}u_* = -\, ½\ \kappa\ \frac{(GM_o)^{1/2}}{(R\rho)^{1/2}}\left(1 + \frac{M}{M_o}\right)^{1/2} \qquad (28)$$

where from (16), $M = mR$. In the discrete system, since state (B) is mass-free, (21) is unchanged.

3.4 The Planetary System

The Planetary System offers an excellent laboratory for examining the conclusions reached in Section 3.3 with the transformation that for a spherical planetary body of mass (M_P) and of diameter D, $W = D$, and the annular mean density, $\rho = 2\, M_P/(\pi^2\, D^2 R)$.

Firstly, from Table 1, based on Jones [8], the sum of the planetary masses, $M = 3\, 10^{27}$ kg, and the sun's mass, M_o is $2\, 10^{30}$ kg, from which $M/M_o \leq 1.5\, 10^{-3}$, and there are two groups of planets (Venus and Earth) and (Uranus and Neptune) within which ρR, and D, m and ρ are approximately constant, and significantly ρR is almost equal between the two groups (which correspond respectively with the terrestrial and the icy planets), in agreement with the Newtonian prediction (14).

We have evaluated (27) for Venus - Earth, and for Uranus-Neptune using Table 1, and find respectively that $K = -5.91\ 10^{-4}$ and $-1.07\ 10^{-3}$. These values are in stark contrast to the balance of forces occurring between Earth-Mars, and Mars-Jupiter, where, from Table 1, on using the average annular density of each pair of planets, as in the inertial coupling model (I), which are respectively, $\rho = 0.0311$ and 0.0183 kgm^{-3}, we find that $K = 2.30\ 10^{-2}$ and $-1.60\ 10^{-2}$.

The Newton gravitational force is emphatically dominant for Venus-Earth and for Uranus-Neptune, but the discrete Prandtl adjunct gravitational force is a significant player in the Earth-Mars-Jupiter environment, where on averaging the estimates for each side of the Martian orbit, it is 1.95 % of the Newton force. The opposite signs on the two sides indicate that the Prandtl force causes a deflection of the gravitational vector of 1.1°. For Earth, we have an imbalance of interplanetary forces in which that due to Venus, $K = -\,5.91\ 10^{-4}$, and that due to Mars, $K = 2.30\ 10^{-2}$, giving rise to a gravitational 'wobble'. These predictions would appear to be able to be compared with observations through an experiment on Mars, or a high precision experiment on Earth, in which the gravitational vector is monitored during the orbit of the planets around the Sun.

4. CONCLUSIONS

The principal conclusion of the analysis is that there are two forces at play, the Newton Force and the Prandtl Force. The application of the analysis to the Planetary System gives a new dimension of understanding in terms of interplanetary stresses (Section 3.4), and arguably of our special place in the scheme of things. The absence of shear stress is not only seminal for the force of gravity, but also, through (14) for the structure of the planetary system, regarded as a continuum.

The magnitude of the Prandtl forces, and of the associated shear stresses is proportional to the square of von Karman's constant (κ). We have used the Earth-bound, experimentally determined value of 0.4 in the numerical estimates. This value is predicted by the formula,

$$\kappa = (2\pi)^{-1/2} \tag{29}$$

(Baumert 2009) in a primitive turbulence analysis, however no convincing evidence for the validity of (29) herein, has yet been found. The philosophical question is whether κ is a fundamental constant of nature, which is applicable on the cosmological scale of this analysis, and also on the meso-scale of fluid boundary layers.

With respect, we propose that the circumferential Prandtl force, which is the adjunct force of gravity, as defined by (26) for the two-body system in which the azimuthal stress, $\tau_{R\phi}$, is given by (24) be included in cosmological modelling.

There are two underlying principles, which are: (i) mass conservation and, (ii) the inertial coupling of two fluids of large density contrast, in the application of which, the annular density (ρ) of the orbiting material is used as a variable, in a look-alike manner to a fluid. There is no direct reference to the General Theory of Relativity, rather our arguments centre around the concept of stress within the gravitational framework, through the Prandtl model.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX TORQUE IN THE EINSTEIN MODEL

The application of the Prandtl model to the Einstein model is straightforward. For $M \gg M_o$, $A^2 = GM/4\pi^2$ and on evaluating (12), we obtain,

$$u_* = -\tfrac{1}{2}\kappa \left(\frac{GM}{R}\right)^{1/2} \frac{d(R\rho)}{dR} \tag{A1}$$

where $U(R) = c$ in which $c = \pm\left(\frac{GM}{R}\right)^{1/2}$ is the speed of light. Since from (14),

$$\rho(R) = \text{constant} \tag{A2}$$

(A1) reduces to,

$$u_* = \pm\tfrac{1}{2}\kappa c \tag{A3}$$

and hence,

$$\tau_{R\phi} = \pm ¼\rho\kappa^2 c^2 \quad (A4)$$

from which, on assuming an equal likelihood for a positive and negative speed of light, the mean azimuthal stress is zero, as assumed in Section 3.3 (a) and the standard deviation of the azimuthal stress,

$$T_{R\rho} = ¼\rho\kappa^2 c^2 \quad (A5)$$

On substituting $\tau_{R\phi} = T_{R\phi}$ from (A5), in (24), and evaluating, dJ, we obtain,

$$dJ = -¼\,\kappa^2 \left(\frac{GM}{R}\right) \rho W dR \quad (A6)$$

and

$$dF = -\left(\frac{GM_o}{R}\right) 2\pi\rho W dR \quad (A7)$$

and hence,

$$\frac{dJ}{dF} = \frac{\kappa^2}{8\pi}\left(\frac{M}{M_o}\right) \quad (A8)$$

which, for $M/M_o \gg 1$, shows the dominance of the Prandtl force over the Newton force for any discrete body (M_o), or in generic terms, of the universal Einstein adjunct force of gravitation over the Newton force of gravitation. On defining the differential torque by the relation, $dQ = RdJ$, and substituting for dJ from (A6) in which $GM/R = c^2$, using mass conservation, we obtain,

$$dQ = ¼\,\kappa^2 \left(\frac{m\,c^2}{2\pi}\right) dR \quad (A9)$$

which on integration yields,

$$Q = \frac{\kappa^2}{8\pi} M\,c^2 \quad (A10)$$

Eq. (A10) is the companion relation for the standard deviation of torque (Q) to Einstein's famous law (20) for energy (E). In the Einstein model, torque and energy are in the ratio, $\kappa^2/8\pi$.

The uncertainty in (A5) lies with the estimate of von Karman's constant. In the numerical evaluations, we use the Earth-bound experimental value, $\kappa = 0.4$ [7]. By the use of $\kappa = 0.4$, we are essentially proposing that κ is a universal quantity.

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