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Solar Radiation Pressure Effects on Stability of Periodic Orbits in Restricted Four-Body Problem

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Abstract

In this work, the Hamiltonian of the four-body problem is considered under the effects of solar radiation pressure. The equations of motion of the infinitesimal body are obtained in the Hamiltonian canonical form. The libration points and the corresponding Jacobi constants are obtained with different values of the solar radiation pressure coefficient. The motion and its stability about each point are studied. A family of periodic orbits under the effects of the gravitational forces of the primaries and the solar radiation pressure are obtained depending on the pure numerical method. This purpose is applied to the Sun-Earth-Moon-Space craft system, and the results obtained are in a good agreement with the previous work such as (Kumari and Papadouris, 2013).

Keywords

Restricted Four-Body Problem, Poincare Surface Sections, Libration Points, Stability, Periodic Orbits

1. Introduction

Since the restricted four-body problem has an important role in astro-dynamics and space dynamics, therefore many attempts dealing with this problem have been done using numerical and analytical methods. Numerical simulations of the one-dimensional Newtonian four-body problem for the special case were conducted in which the bodies are distributed symmetrically about the centre of mass and were studied the Schubart-like periodic orbit's stability to perturbation where it is apparently stable in one-dimension but is unstable in three-dimensions [1]. The effect of radiation on some dynamical system of four-body problem was studied to obtain the location and stability of Lagrangian points [2]. Spatial equilateral restricted four-body problem was studied, obtained a first integral of motion with the help of the Hamiltonian structure, and showed the existence of periodic solutions by different methods in the planar case [3]. The families of simple symmetric and non-symmetric periodic orbits in the restricted four-body problem were presented [4]. Symmetric periodic orbits of the restricted four-body problem for the case of two equal masses were explored where they satisfy approximately the Routh's critical value [5]. The zero-velocity curves of the four-body problem were studied with solar wind drag [6]. The photo gravitational version of the problem of four bodies was studied numerically where an infinitesimal particle is moving under the Newtonian gravitational attraction of three bodies which are finite moving in circles around their center of mass fixed at the origin of the coordinate system, according to the solution of Lagrange where they are always at the vertices of an equilateral triangle [7]. An analytical study of the elliptic Sitnikov restricted four-body problem was presented when all the primaries considered as source of same radiation pressure [8]. The network of the families of simple symmetric periodic solutions of the restricted four-body problem was investigated and the effect of radiation on the periodic orbits was studied, their stability, as well as the evolution of the families when the radiation parameter varies, and Poincare sections of the problem are illustrated [9]. The classical lunar Hill problem was extended and the geometry of Poincare sections was investigated, also the direct and retrograde periodic orbits about the infinitesimal mass and their stable and unstable manifolds were studied [10]. Periodic orbits in the photo gravitational restricted problem when the primaries are tri-axial rigid bodies were investigated [11]. The Effect of Solar Radiation Pressure on the Libration Points of the Restricted Four-Body Problem was studied and the periodic orbits were presented [12]. The restricted four-body problem was studied and the orbits which emanated from some equilibrium points in which they focused on some families of symmetric horseshoe orbits were investigated and their relation with a family of the Lyapunov orbits was shown [13]. The photo-gravitational restricted four-body problem was studied with variable mass, zero velocity curves and Newton-Raphson basins of attraction were also discussed [14]. The existence of collinear and non-collinear equilibrium points and their linear stability in the framework of photo gravitational circular restricted four-body problem with Stokes drag acting as a dissipative force and the first primary as a radiating body and the second primary as an oblate spheroid was studied numerically [15].

In this work the Hamiltonian of the restricted four-body problem is constructed under the effect of solar radiation pressure, the location of the libration points are obtained at different values of the solar radiation pressure coefficient, the stability of motion about the collinear libration points is studied and Poincare surface sections is used to illustrates these stabilities.

2. Solar Radiation Pressure

According to Newton-Lebedev law, the general photo-gravitational force is described as the geometrical sum of two opposite forces, 1) Apart from the gravitational acceleration of the Sun on the spacecraft $F_g = \frac{Gm_s m}{r_s^2}$, m refers to the mass of space craft, while m_s refers to the mass of Sun; 2) The radiation force acting on spacecraft is obtained by $F_{rad} = \frac{L_\odot}{4\pi r_s^2 c} A$, $L_\odot$ is Luminosity of Sun, A is the cross-section area of the spacecraft surface, c is the speed of light, and r_s is the distance between Sun and the spacecraft. So that

$$F = F_g - F_{rad} = F_g\left(1 - \frac{F_{rad}}{F_g}\right) = F_g(1-\beta) \tag{1}$$

where, $\beta = \frac{F_{rad}}{F_g} = \frac{L_\odot}{4\pi r_s^2 c}\frac{1}{Gm_s}\frac{A}{m}$

The potential of the force exists by the Sun is

$$V_{RP} = -\int F\mathrm{d}r = -\int F_g(1-\beta)\mathrm{d}r = -(1-\beta)\int\frac{m_s}{r_s^2}\mathrm{d}r = (1-\beta)\frac{m_s}{r_s} \tag{2}$$

Then, in the case of restricted four-body problem the effective potential included the effects of solar radiation pressure (SRP) is given by

$$V(x,y,z) = \frac{1}{2}\left(x^2+y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\beta)\mu_s}{r_3} \tag{3}$$

where x, y, z are the coordinates of the fourth body, r_1, r_2, r_3 are the dimensionless distances from the fourth body to the primaries, and $\mu, 1-\mu, \mu_s$ are the dimensionless masses for the primaries, defined as

$$\mu = \frac{m_2}{m_1+m_2}; \quad \mu_s = \frac{m_s}{m_1+m_2} \tag{4}$$

where m_1, m_2 and m_s are the masses of primaries respectively.

3. The Hamiltonian System of RFBP with (SRP)

To construct the Hamiltonian of the restricted four-body problem using the concept of Lagrange and Hamiltonian principle for the rotating frame (ξ,η,ζ)

$$L\left(\xi,\eta,\zeta,\dot{\xi},\dot{\eta},\dot{\zeta}\right) = T - V = \frac{1}{2}\left(\dot{\xi}^2+\dot{\eta}^2+\dot{\zeta}^2\right) + G\left[\frac{m_1}{r_1} + \frac{m_2}{r_2} + (1-\beta)\frac{m_3}{r_3}\right] \tag{5}$$

Since

$$\xi = x\cos\omega + y\sin\omega \tag{6.1}$$

$$\eta = x\sin\omega - y\cos\omega \tag{6.2}$$

$$\zeta = z \tag{6.3}$$

where, $\omega = nt$ is the angular velocity of the rotating system, ω is the rate at which the primaries rotates about their center of mass and change their position per time (second) and n is the mean motion. Using Equation (3) and Equation (6), after some little algebraic reductions then

$$L(x,y,z,\dot{x},\dot{y},\dot{z})=\frac{1}{2}\left[\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)+\left(x^2+y^2\right)+2\left(x\dot{y}-\dot{x}y\right)\right] +\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{(1-\beta)\mu_s}{r_3} \tag{7}$$

By using the definition of the momentum $p_i=\frac{\partial L}{\partial \dot{q}_i}$, then

$$\dot{x}=p_x+y \tag{8.1}$$

$$\dot{y}=p_y-x \tag{8.2}$$

$$\dot{z}=p_z \tag{8.3}$$

the Hamiltonian is defined by [16]

$$\mathcal{H}=\sum_i p_i\dot{q}_i-L \tag{9}$$

Substitute from Equation (7) and Equation (8) into Equation (9), this yields the Hamiltonian of the restricted four-body problem with solar radiation pressure in the form

$$\mathcal{H}=\frac{1}{2}\left(p_x^2+p_y^2+p_z^2\right)+yp_x-xp_y-\left[\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+(1-\beta)\frac{\mu_s}{r_3}\right] \tag{10}$$

where

p_x, p_y, p_z are the components of momenta in Cartesian coordinates. And the canonical form is given by

$$\dot{x}=\frac{\partial\mathcal{H}}{\partial p_x}=p_x+y \tag{11.1}$$

$$\dot{y}=\frac{\partial\mathcal{H}}{\partial p_y}=p_y-x \tag{11.2}$$

$$\dot{z}=\frac{\partial\mathcal{H}}{\partial p_z}=p_z \tag{11.3}$$

$$\dot{p}_x=-\frac{\partial\mathcal{H}}{\partial x}=\dot{y}+x-\frac{(1-\mu)(x+\mu)}{r_1^3}-\frac{\mu(x+\mu-1)}{r_2^3} -\frac{\mu_s(1-\beta)(x-R_s\cos\theta)}{r_3^3} \tag{11.4}$$

$$\dot{p}_y=-\frac{\partial\mathcal{H}}{\partial y}=-\dot{x}+y-\frac{(1-\mu)\cdot y}{r_1^3}-\frac{\mu y}{r_2^3}-\frac{\mu_s(1-\beta)(y-R_s\sin\theta)}{r_3^3} \tag{11.5}$$

$$\dot{p}_z=-\frac{\partial\mathcal{H}}{\partial z}=-\frac{(1-\mu)\cdot z}{r_1^3}-\frac{\mu z}{r_2^3}-\frac{\mu_s(1-\beta)z}{r_3^3} \tag{11.6}$$

where

$\theta=\omega_s t$, ω_s is the angular velocity of the center of mass of the two primaries about the Sun, and R_s is the distance between the Sun and the center of mass of the two primaries. Equation (11) represents the equations of motion of the fourth body under the effect of gravitational forces and the solar radiation pressure.

Now, the Jacobi constant is defined as

$$\dot{p}_x^2 + \dot{p}_y^2 + \dot{p}_z^2 = -2V + C \tag{12}$$

when the velocity tends to zero, then Equation (12) becomes

$$C = 2V \tag{13}$$

Equation (13) enables to obtain the zero velocity curves and is used to apply the Poincare surface sections (PSS) to study the stability of motion about each libration point.

4. Location of the Libration Points with Effect of SRP

One of the special solutions of the restricted four-body problem is the equilibrium points at which the components of the velocity of the fourth body are zero. The subject of equilibrium points is to find the location of the points where a fourth body could be placed. To obtain the location of libration points, the conditions $\dot{x} = \dot{y} = \dot{z} = \dot{p}_x = \dot{p}_y = \dot{p}_z = 0$, are applied on Equation (11), then

$$p_x + y = 0 \tag{14.1}$$

$$p_y - x = 0 \tag{14.2}$$

$$p_z = 0 \tag{14.3}$$

Then

$p_x = -y$, $p_y = x$, and $p_z = 0$, then

$$x - \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x+\mu-1)}{r_2^3} - \frac{\mu_s(1-\beta)(x - R_s\cos\theta)}{r_3^3} = 0 \tag{15.1}$$

$$y - \frac{(1-\mu)\cdot y}{r_1^3} - \frac{\mu y}{r_2^3} - \frac{\mu_s(1-\beta)(y - R_s\cos\theta)}{r_3^3} = 0 \tag{15.2}$$

$$-\frac{(1-\mu)\cdot z}{r_1^3} - \frac{\mu z}{r_2^3} - \frac{\mu_s(1-\beta)z}{r_3^3} = 0 \tag{15.3}$$

Since the motion lies in the x-y plane then Equation (15.3) will be vanished. Now, the collinear points can be determined from Equation (15.1), with $\theta = 0$

$$x = \frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x+\mu-1)}{r_2^3} - \frac{\mu_s(1-\beta)(x - R_s)}{r_3^3} \tag{16}$$

Let X_L denotes the coordinates of the libration points, then applying the binomial function on Equation (16), then

$$\begin{aligned} &X_L^5\left(R_s^3 - \mu_S(1-\beta)\right) + X_L^4\left(-2R_s^3 + 2\mu_S(1-\beta) + \mu_S R_s(1-\beta)\right. \\ &\left.+4R_s^3\mu - 4\mu_S(1-\beta)\mu\right) + X_L^3\left(R_s^3 - \mu_S(1-\beta) - 2\mu_S R_s(1-\beta)\right. \\ &\left.-6R_s^3\mu + 6\mu_S(1-\beta)\mu + 4\mu_S R_s(1-\beta)\mu + 6R_s^3\mu^2 - 6\mu_S(1-\beta)\mu^2\right) \\ &+X_L^2\left(-R_s^3 + \mu_S R_s(1-\beta) + 2R_s^3\mu - 2\mu_S(1-\beta)\mu - 6\mu_S R_s(1-\beta)\mu\right. \end{aligned}$$

$$
\begin{aligned}
&-6R_s^3\mu^2+6\mu_S\left(1-\beta\right)\mu^2+6\mu_S R_s\left(1-\beta\right)\mu^2+4R_s^3\mu^3-4\mu_S\left(1-\beta\right)\mu^3\Big)\\
&+X_L\Big(2R_s^3-4R_s^3\mu+2\mu_S R_s\left(1-\beta\right)\mu+R_s^3\mu^2-\mu_S\left(1-\beta\right)\mu^2\\
&-6\mu_S R_s\left(1-\beta\right)\mu^2-2R_s^3\mu^3+2\mu_S\left(1-\beta\right)\mu^3+4\mu_S R_s\left(1-\beta\right)\mu^3\\
&+R_s^3\mu^4-\mu_S\left(1-\beta\right)\mu^4\Big)+3R_s^3\mu-3R_s^3\mu^2+\mu_S R_s\left(1-\beta\right)\mu^2\\
&-2\mu_S R_s\left(1-\beta\right)\mu^3+\mu_S R_s\left(1-\beta\right)\mu^4-R_s^3=0
\end{aligned}
\tag{17}
$$

This is a quantic equation and its solution has five real parts depends on the parameters μ_S,μ,R_s and β these roots give the positions of the collinear libration points.

5. Motion in the Vicinity of the Collinear Libration Points and Its Stability

The Sun-Earth-Moon system is considered with the effect of SRP. Then the perturbed motion around collinear libration points is obtained with initial conditions of small displacement from x_0, y_0 and $\dot{x}_0=\dot{y}_0=0$. To obtain the periodic orbits family about each of collinear libration points for different values of β the following steps are used.

1) The values of potentials V_{xx},V_{yy} are determined for the value of β and the corresponding coordinate of libration point.

2) The Eigen values are obtained from the characteristic equation.

3) The Eigen values will be four values (2 real and 2 imaginaries).

4) The two imaginary values responding to give the stable periodic orbits about the libration point.

Now, to apply these steps the linear equations for the motion about the collinear libration points for the fourth body are written as follows,

$$\dot{p}_x-2p_y=xV_{xx}+yV_{xy}+zV_{xz}, \tag{18.1}$$

$$\dot{p}_y+2p_x=xV_{xy}+yV_{yy}+zV_{yz}, \tag{18.2}$$

$$\dot{p}_z=zV_{zz}. \tag{18.3}$$

where the partial derivatives of the effective potential for the four-body problem with SRP are obtained as

$$
\begin{aligned}
V_{xx}=1&+\frac{3m_s\left(-R_s+x\right)^2\left(1-\beta\right)}{\left(\left(-R_s+x\right)^2+y^2+z^2\right)^{5/2}}-\frac{m_s\left(1-\beta\right)}{\left(\left(-R_s+x\right)^2+y^2+z^2\right)^{3/2}}\\
&+\frac{3\mu\left(-1+x+\mu\right)^2}{\left(y^2+z^2+\left(-1+x+\mu\right)^2\right)^{5/2}}-\frac{\mu}{\left(y^2+z^2+\left(-1+x+\mu\right)^2\right)^{3/2}}\\
&+\frac{3\left(1-\mu\right)\left(x+\mu\right)^2}{\left(y^2+z^2+\left(x+\mu\right)^2\right)^{5/2}}-\frac{1-\mu}{\left(y^2+z^2+\left(x+\mu\right)^2\right)^{3/2}}
\end{aligned}
\tag{19.1}
$$

$$V_{yy}=1+\frac{3m_s y^2(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{5/2}}-\frac{m_s(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{3/2}}+\frac{3y^2\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{5/2}}-\frac{\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{3/2}}+\frac{3y^2(1-\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{5/2}}-\frac{1-\mu}{\left(y^2+z^2+(x+\mu)^2\right)^{3/2}} \tag{19.2}$$

$$V_{zz}=1+\frac{3m_s z^2(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{5/2}}-\frac{m_s(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{3/2}}+\frac{3z^2\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{5/2}}-\frac{\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{3/2}}+\frac{3z^2(1-\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{5/2}}-\frac{1-\mu}{\left(y^2+z^2+(x+\mu)^2\right)^{3/2}} \tag{19.3}$$

$$V_{xy}=V_{yx}=\frac{3m_s(-R_s+x)y(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{5/2}}+\frac{3y\mu(-1+x+\mu)}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{5/2}}+\frac{3y(1-\mu)(x+\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{5/2}} \tag{19.4}$$

$$V_{zy}=V_{yz}=\frac{3m_s yz(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{5/2}}+\frac{3yz\mu}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{5/2}}+\frac{3yz(1-\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{5/2}} \tag{19.5}$$

$$V_{zx}=V_{xz}=\lambda\frac{3m_s(-R_s+x)z(1-\beta)}{\left((-R_s+x)^2+y^2+z^2\right)^{5/2}}+\frac{3z\mu(-1+x+\mu)}{\left(y^2+z^2+(-1+x+\mu)^2\right)^{5/2}}+\frac{3z(1-\mu)(x+\mu)}{\left(y^2+z^2+(x+\mu)^2\right)^{5/2}} \tag{19.6}$$

Then characteristic equation can be rewritten as

$$\lambda^4+\left(4-V_{xx}-V_{yy}\right)\lambda^2+V_{xx}V_{yy}=0 \tag{20}$$

The solutions of the linear Equation (18) can be written as

$$x=A_i\sum_{i=1}^{4}e^{\lambda_i t} \tag{21.1}$$

$$y=B_i\sum_{i=1}^{4}e^{\lambda_i t} \tag{21.2}$$

where, A_i and B_i represent constant coefficient. Equation (20) has four roots $\lambda_i, i=1,2,3,4$ the real roots of λ_i give unstable motion, while the imaginary roots represent the stable motion.

$$e^2 = \frac{d^2 - 1}{d^2} \tag{22.1}$$

$$T = \frac{2\pi}{|s|} \tag{22.2}$$

where

$d = \frac{\lambda_i^2 - V_{xx}}{2\lambda_i - V_{xy}}$, and s is the coefficient of the imaginary Eigen value.

6. Results and Discussion

The Sun-Earth-Moon-spacecraft system is used to illustrate this work. The Earth's mass $m_1 = 5.98 \times 10^{24}\,\text{kg}$, the mass of Moon $m_2 = 7.35 \times 10^{22}\,\text{kg}$, the mass of Sun $m_s = 1.99 \times 10^{30}\,\text{kg}$, Reena Kumari (2013). The canonical units of masses and distances are used in which the mass of the Earth

$= \mu_E = 1 - \mu = \frac{m_1}{m_1 + m_2} = 0.9878715$; mass of the Moon

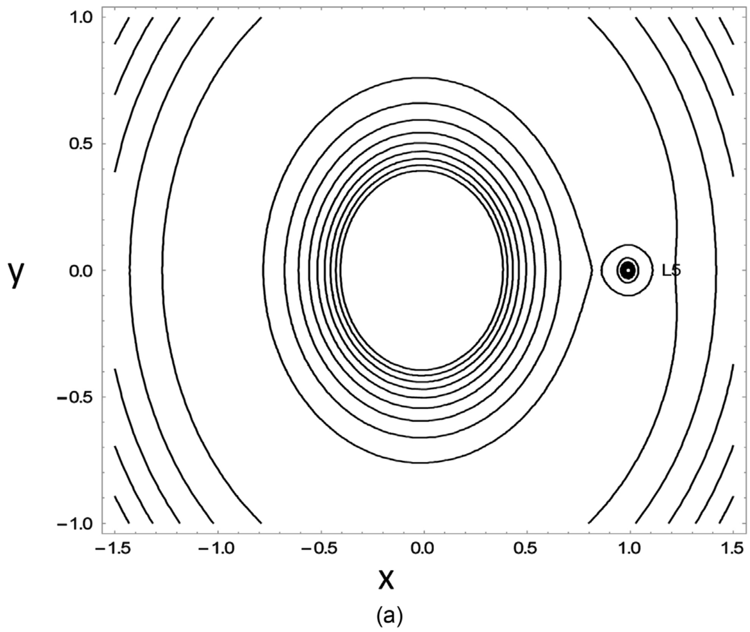
$= \mu_M = \mu = \frac{m_2}{m_1 + m_2} = 0.0121506683$; mass of the Sun

$= \mu_s = \frac{m_s}{m_1 + m_2} = 328900.48$, and the distance between the Sun and the center of the system $= R_s = 389.1723985$.

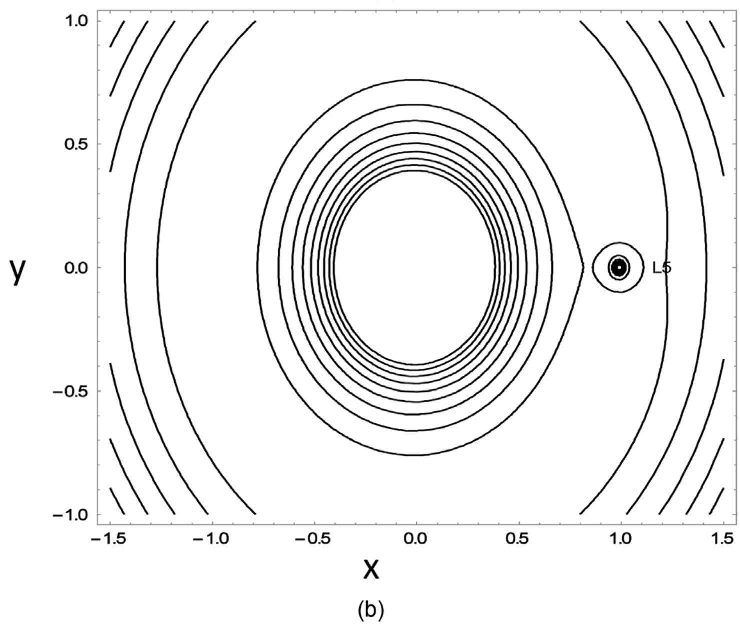
Now, the solution of Equation (17) numerically will gives the locations of the collinear libration points, the corresponding Jacobi constant is obtained from Equation (13), Table 1 shows the collinear libration points and the corresponding Jacobi constant at different values of the solar radiation pressure β, it is notice that there is a shift in positions of each of libration points related to the values of the solar radiation pressure coefficient β. It is clear from Table 1 that L1, L3 and L5 shifted to right of collinear axis with the increasing of solar radiation coefficient β, while L2 and L4 shifted to left with increasing the solar radiation coefficient β. Figure 1(a) and Figure 1(b) illustrates the ZVC at the L5 without SRP and with SRP respectively. When the Jacobi constant C is large, the separated

Table 1. The five collinear libration points for the Sun-Earth-Moon system and their Jacobi constants C with different values of β.

β	L1		L2		L3		L4		L5	
	X	C	X	C	X	C	X	C	X	C
0	−1.86027	7.87523	−0.89726	4.09822	0.59344	2.49354	0.90838	4.40506	1.05927	8.82768
0.01	−1.87369	7.87823	−0.90095	4.05882	0.59680	2.46279	0.90755	4.34356	1.05984	8.77038
0.03	−1.80859	7.49129	−0.92646	4.05485	0.60150	2.38657	0.90653	4.22586	1.06049	8.65799
0.05	−1.73955	7.09377	0.95601	4.06534	0.60631	2.31073	0.90547	4.10788	1.06116	8.54558
0.07	−1.66483	6.67933	−0.99133	4.09659	0.61124	2.23536	0.90436	3.98963	1.06184	8.43319
0.09	−1.58098	6.23529	−1.0359	4.16138	0.61629	2.16043	0.90320	3.87109	1.06254	8.32077
0.1	−1.533	5.99366	−1.06394	4.21476	0.61886	2.12312	0.90260	3.81169	1.0629	8.26455



(a)



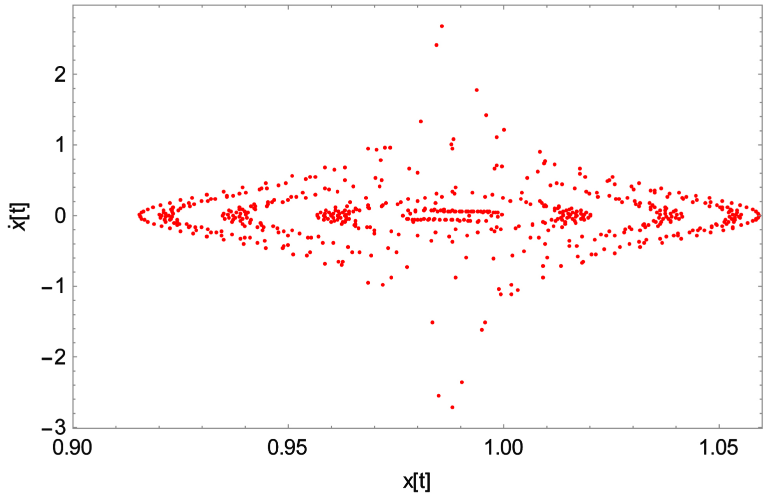
(b)

Figure 1. (a): Zero velocity curves without SRP. (b): Zero velocity curves with SRP.

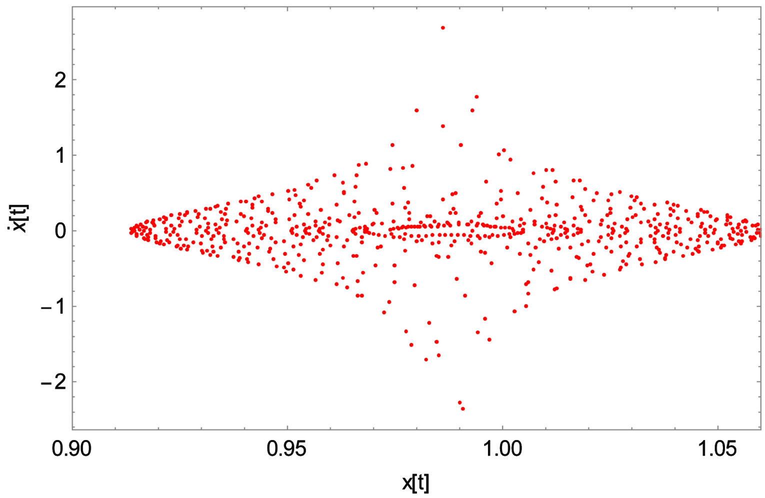
areas are allowed at which the fourth body is moving and never moves from one allowed region to another.

Also, the effects of SRP separate the asymptotic circles. Figure 2(a) and Figure 2(b) show the surface of sections about L5 without the effects of SRP and with the effects of SRP respectively.

Figure 2(a) and Figure 2(b) are obtained by using a cod of Mathematica version 10 to solve the Equation (11) numerically and by using the event locator method to illustrate that Figure 2(a) shows the regular islands without SRP, while in Figure 2(b) the regular islands with the radiation pressure are shown,



(a)



(b)

Figure 2. (a): The surface of sections about L5 without SRP. (b): The surface of sections about L5 with SRP.

they are more closed around the libration point (the center). Here, the regular islands are expanded gradually because of radiation pressure. Again, the island centered about $x = 1.06$ shows that the trajectory is regular, which mean that the region in the neighborhood of $x = 1.06$ is stable, and this region shrinks towards center. This is more clear at **Figure 3(a)** and **Figure 3(b)** at which the Poincare surface of sections are presented, which are projections of the trajectories on the (x-$\dot{x}$) plane and that means there is a periodic orbit at each point of the projection and it is clear that they are concentrated at the center.

The family of periodic orbits about L5 which related to different values of solar radiation pressure coefficient β is shown in **Figure 4**, their eccentricities and periodic orbits are obtained by using Equation (22), the inner orbit for the smaller value of β, as the value of β increases the orbit is bigger. This depends on the calculations of $d = \dfrac{\lambda_i^2 - V_{xx}}{2\lambda_i - V_{xy}}$, which specify the eccentricity of the orbit, it is

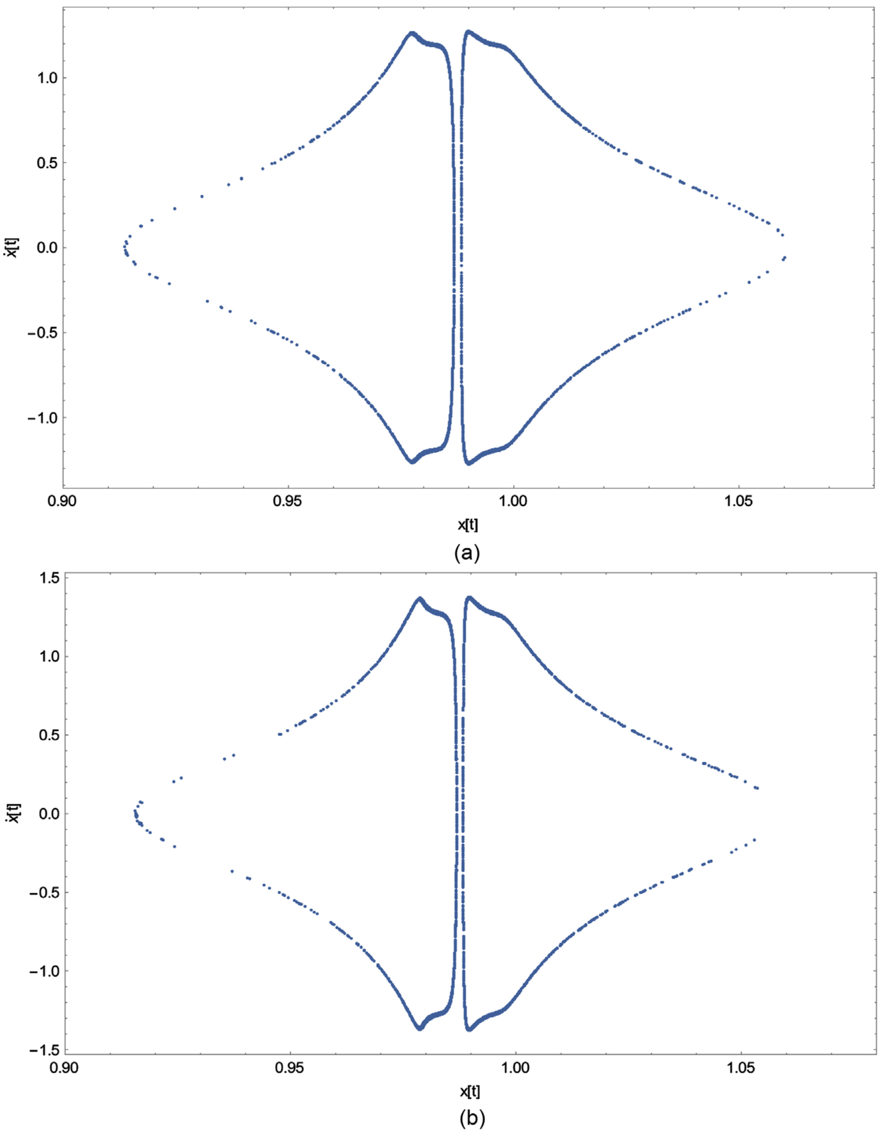


Figure 3. (a): PSS without SRP. (b): PSS with SRP.

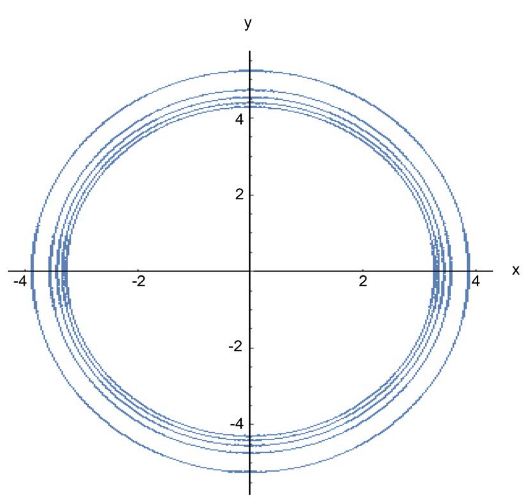


Figure 4. Families of periodic orbits around the point L5.

clear from Equation (19) that the values of β has a great effects on the eccentricity of the orbit.

7. Conclusion

In this work, the study is concentrated on the motion about the collinear libration points. The restricted four-body problem is studied by assuming the effect of radiation pressure. The boundaries of allowed regions for the motions of the infinitesimal mass are determined using zero velocity surfaces at different values of the radiation pressure coefficient. It is found that allowed possible regions of the motions decrease with the increase in the value of Jacobi constant C. With the help of PSS, it is observed that the stability region gets expanded in presence of radiation pressure and at the point $x = 1.0629$ orbits are stable. The effect of solar radiation pressure controls the positions of the libration points and the stability of motion about these libration points. This work enables the maneuvers to be done in the spacecraft missions.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Nomenclature

F_g : Gravitational force,
F_{rad} : Radiation force,
m: Mass of spacecraft,
m_s: Mass of the Sun,
m_1 : Mass of the Earth,
m_2 : Mass of the Moon,
r_s : The dimensionless distance between Sun and the spacecraft,
r_1 : The dimensionless distance between Earth and the spacecraft,
r_2 : The dimensionless distance between Moon and the spacecraft,
$L_{\odot}$: Luminosity of Sun,
A: The cross-section area of the spacecraft surface,
c: The speed of light,
β : The solar radiation pressure parameter,
V_{RP} : The potential of solar radiation pressure force,
μ : The dimensionless mass of Moon,
$1-\mu$: The dimensionless mass of Earth,
μ_s : The dimensionless mass of Sun,
ω : The angular velocity of the rotating system,
n: The mean motion,
p_x, p_y, p_z : The components of momentum in Cartesian coordinates,
ω_s : The angular velocity of the center of mass of the two primaries about the Sun.