



# **Parameter Estimation of Three-Parameter Weibull Distribution of Gas Turbine Blades Using TPC Windchill Quality Solutions**

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## **Authors' contribution**

*This work was carried out in collaboration among all authors. Author ET designed the study, performed the statistical analysis and wrote the first draft of the manuscript. Authors WEO and JEU managed the analyses of the study. All authors read and approved the final manuscript.*

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## **ABSTRACT**

The maximum likelihood estimation method is an effective technique for estimating the parameters of the Weibull distribution. However, it is an arduous task to compute the parameters of the Weibull distribution using numerical methods, hence; various reliability software packages have been developed to address this difficulty. In this study, an attempt is made to obtain the estimates of three-parameter Weibull distribution through the application of Weibull analysis software TPC Windchill Quality Solutions 11. The study involves the analysis of the failure times of ten identical gas turbine blades over a period of ten years. From the results obtained, it was found that the gas turbine blades were in their wear-out period. The results obtained in the study were compared with Weibull analysis software Minitab 19 and the values of the Weibull estimates obtained were found to be close. This shows that the software is suitable for the parameter estimation of three-parameter Weibull distribution.

**Keywords:** *Failure time; reliability; maximum likelihood estimation; three-parameter Weibull; TPC Windchill Quality Solutions; probability plots; gas turbine blades.*

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## 1. INTRODUCTION

In the field of engineering, one of the most widely used distributions in reliability and life data analysis due to its versatility is the Weibull distribution [1]. It is used to study the downtime of equipment or parts of equipment, for fitting engineering data, such as the strength of materials [2,3], fracture of brittle materials [4], and wind speed [5,6]. The three-parameter Weibull distribution is very suitable for fitting random data due to its strong flexibility to various types of probability distribution. It is very important that the estimates of the three-parameter Weibull are determined correctly in order to fully apply the Weibull method. There are several graphical and analytical methods for estimating the values of these parameters. The graphical methods include Weibull probability plot (WPP) and hazard plot while the analytical methods include maximum likelihood method (MLE), least square method and method of moments. The analytical methods are considered to be more reliable and accurate than the graphical methods. From literature, the three-parameter Weibull distribution parameters estimation is a very complex procedure. To date, several estimation methods have been conducted, such as graphic method [7,8], moment estimation [9], maximum likelihood estimation (ML) [10–12], kernel density estimation [13,14], least squares estimation with particle swarm optimization [15] etc. Ahmad et al. [16] compared the three well known methods (maximum likelihood method, method of moment, least square method) of parameter estimation of the Weibull distribution using the two-parameter and three-parameter Weibull distributions. Li [17] proposed a method that depends on two-step iteration procedure. The first step assumed that the location parameter was known while the scale and shape parameters were estimated using the graphical methods. In the second step, the shape parameter was fixed while the scale and location parameter were estimated by transformation of the data by using the transformation law so that the transformation data can be modeled to a two-parameter exponential distribution. Yang et al. [18], Dai et al. [19], and Shi et al. [20] presented an advanced method for maximum likelihood estimation of three-parameter Weibull distribution. Cohen [21] described the maximum likelihood estimation method and derived the covariance matrix of the parameter of a two-parameter Weibull distribution based on

censored and complete sample. Harter and Moore [22] developed an iterative method to determine the maximum likelihood estimate for a three-parameter Weibull distribution and showed numerical examples for; one-parameter, two-parameter, and three-parameter Weibull model. In this study, consideration was done for maximum likelihood estimation using reliability software Windchill Quality Solutions to obtain estimates of three-parameter Weibull distribution.

## 2. METHODOLOGY

The data used in this study was obtained from maintenance records in an oil and gas industry located in the South-South region of Nigeria for a period of ten years. The data consists of times to failure of ten identical gas turbine blades. The data gathered was analyzed and used in the modeling done in this work. Assurance was given to the organization during data collection that the data would be kept confidential and used only for academic purposes. The Weibull method is a very significant life testing model which provides the generalization of many other life testing distributions such as the two-parameter exponential distribution (when the shape parameter  $\beta = 1$ ), two-parameter Weibull distribution (when the location parameter  $\gamma = 0$ ), one-parameter exponential distribution (when the shape parameter  $\beta = 1$  and the location parameter  $\gamma = 0$ ) and a Rayleigh distribution (when the shape parameter,  $\beta = 2$ ). The failure model used in this study is the three-parameter Weibull distribution by applying the maximum likelihood estimation.

### 2.1 Weibull Shape Parameter, $\beta$

The Weibull shape parameter,  $\beta$ , is also known as the Weibull slope. This is because the value of  $\beta$  is equal to the slope of the line in a probability plot. Different values of the shape parameter can have marked effects on the behavior of the distribution. In fact, some values of the shape parameter will cause the distribution equations to reduce to those of other distributions. Depending on the value of the shape parameter, a variety of behaviors can be described:

- i. When the shape parameter,  $\beta < 1$ , the failure rate decreases with time;
- ii. When the shape parameter,  $\beta = 1$ , the failure rate is constant with time and the distribution is equal to the exponential distribution.

- iii. When the shape parameter,  $\beta > 1$ , the failure rate increases with time.

### 2.2 Weibull Scale Parameter, $\eta$

A change in the scale parameter,  $\eta$ , has the same effect on the distribution as a change of the abscissa scale. Increasing the value of  $\eta$  while keeping  $\beta$  constant has the effect of stretching out the probability density function (pdf). Since the area under a pdf curve is a constant value of one, the "peak" of the pdf curve will also decrease with the increase of  $\eta$ .

### 2.3 Weibull Location Parameter, $\gamma$

The location parameter,  $\gamma$ , is the subtracted (positive or negative) value that places the points in an acceptable straight line. Changing the value of the location parameter,  $\gamma$ , has the effect of pushing the distribution and its associated function either to the right (if  $\gamma > 0$ ) or to the left (if  $\gamma < 0$ ).

- i. When the location parameter,  $\gamma = 0$ , the distribution starts at the origin or at time  $t = 0$ .
- ii. When  $\gamma > 0$ , the distribution begins at the location  $\gamma$  to the right of the origin.
- iii. If  $\gamma < 0$ , then the distribution starts at the location  $\gamma$  to the left of the origin.
- iv. The location parameter provides an estimate of the earliest time-to-failure of the units under test.
- v. The location parameter must be less than or equal to the first time-to-failure.
- vi. The location parameter may assume all values and provides an estimate of the earliest time at which a failure may be observed. A negative  $\gamma$  may indicate that failures have occurred prior to the beginning of the data collection period for the analysis. For example, failures might have occurred during production, in storage, in transit, during checkout prior to the start of a mission, or prior to actual use.
- vii. The location parameter has the same units as  $t$ , such as hours, miles, cycles, etc. In order to linearize the original data line, the value of  $\gamma$  must be subtracted from each of the points.

### 2.4 Maximum Likelihood Estimation for Three-Parameter Weibull Distribution

The maximum likelihood estimation is a very important and effective approach for parameter estimation.

The maximum likelihood method for three-parameter Weibull distribution is described briefly. Let  $t_1, t_2, \dots, t_n$  be a random sample of size  $n$ ;  $\bar{\theta}$  is noted as the Weibull parameters which are to be estimated; namely  $\bar{\theta} = (\gamma, \eta, \beta)$ . The likelihood function is written as:

$$L = \prod_{i=1}^n f_t(t_i; \bar{\theta}) \tag{1}$$

$$L = \prod_{i=1}^n \frac{\beta}{\eta} \left(\frac{t_i - \gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i - \gamma}{\eta}\right)^\beta} \tag{2}$$

The aim of the estimation is to determine the unknown vector  $\bar{\theta}$  and the three unknown parameters  $\gamma, \eta$  and  $\beta$  by maximizing the likelihood function. It is easier to obtain the maximum by its logarithm since it contains exponential term. The logarithm of the likelihood is given as:

$$\ln[L(t_i, \bar{\theta})] \tag{3}$$

The vector  $\bar{\theta}$  is obtained by maximizing the likelihood function. This is done by setting the partial equations to zero and the following equations are obtained:

$$L_1 = \sum_{i=1}^n \left[ \frac{1}{\beta} + \ln(t_i - \gamma) - \ln \eta - \left(\frac{t_i - \gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i - \gamma}{\eta}\right)^\beta} \right] = 0 \tag{4}$$

$$L_2 = \sum_{i=1}^n \left[ -\frac{\beta}{\eta} + \left(\frac{\beta}{\eta}\right) \left(\frac{t_i - \gamma}{\eta}\right)^\beta \right] = 0 \tag{5}$$

$$L_3 = \sum_{i=1}^n \left[ -\frac{(\beta-1)}{(t_i - \gamma)} + \left(\frac{\beta}{\eta}\right) \left(\frac{t_i - \gamma}{\eta}\right)^{\beta-1} \right] = 0 \tag{6}$$

However, it is well known that to obtain the estimates of the unknown parameters ( $\eta, \beta$  and  $\gamma$ ) by solving the above maximum likelihood equations by conventional numerical methods is an arduous task. Nowadays, there are specialized Weibull analysis software packages such as Weibull++ by ReliaSoft, Minitab, WeibullPro by Isograph, TPC Windchill Quality Solutions, WinSMITH Weibull by Barringer and associates that contain programs to accurately estimate the Weibull parameters and also generate Weibull plots from a given data set. In this study, TPC Windchill Quality Solutions 11.0 software package is used to determine the estimates of three-parameter Weibull distribution ( $\eta, \beta$  and  $\gamma$ ). When the estimates of the three-parameter Weibull distribution are accurately determined, they can be used to assess the reliability, the hazard/failure rate and mean time to failure of the gas turbine blades.

The reliability function of the three-parameter Weibull distribution is given by:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}, \quad t \geq \gamma \quad (7)$$

The three-parameter Weibull failure rate function is given by:

$$\lambda(t) = \frac{\beta}{\eta} \left(\frac{t-\gamma}{\eta}\right)^{\beta-1}, \quad t \geq \gamma \quad (8)$$

The Mean Time to Failure of the three-parameter Weibull distribution is given by:

$$MTTF = \gamma + \eta \Gamma\left(1 + \frac{1}{\beta}\right) \quad (9)$$

Where  $\gamma \geq 0$ , and  $t, \beta, \eta > 0$

$$\Gamma(x) = \int_0^\infty e^{-x} x^{n-1} \quad (10)$$

Where  $\Gamma(x)$  = Gamma Function  
 Where  $\gamma$  = location parameter,  
 $\eta$  = Scale parameter  
 $\beta$  = Shape parameter  
 $t$  = Time

### 3. RESULTS AND DISCUSSION

#### 3.1 Results

Table 1 shows the failure times (hrs) of ten identical gas turbine blades, same make and model, working under the same stresses and conditions. The data is analyzed using specialized Weibull analysis software PTC Windchill Quality Solutions 11 to obtain the three-parameter Weibull estimates, namely, the location, shape and scale parameters and also to produce the Weibull probability plot, PDF plot, reliability plot, failure rate plot and 3D contour

plot as shown in Table 2 and Figs. 1-5. The results obtained in Tab 2 were compared with those obtained using another reliability software Minitab 19 as shown in Table 3.

**Table 1. Failure time of gas turbine blades**

Rank	Failure time, hrs
1	1047
2	1279
3	1340
4	1578
5	1598
6	1749
7	1804
8	1841
9	1847
10	1869
11	1879
12	1890
13	1939
14	1948
15	1949
16	1956
17	1987
18	1995
19	2004
20	2005
21	2047
22	2214
23	2287
24	2435
25	2439
26	2442
27	2581
28	2617
29	2926
30	2978

**Table 2. Three-parameter Weibull estimates using TPC Windchill Quality Solutions 11**

Shape parameter, $\beta$	Scale parameter, $\eta$	Location parameter, $\gamma$
3.163	1407.7766	755.2533

**Table 3. Comparison of results using TPC Windchill Quality Solutions and Minitab**

Weibull estimates	TPC Windchill Quality Solutions	Minitab 19
Location parameter, $\gamma$	755.2533	754.8611
Shape parameter, $\beta$	3.163	3.157
Scale parameter, $\eta$	1407.7766	1408.134

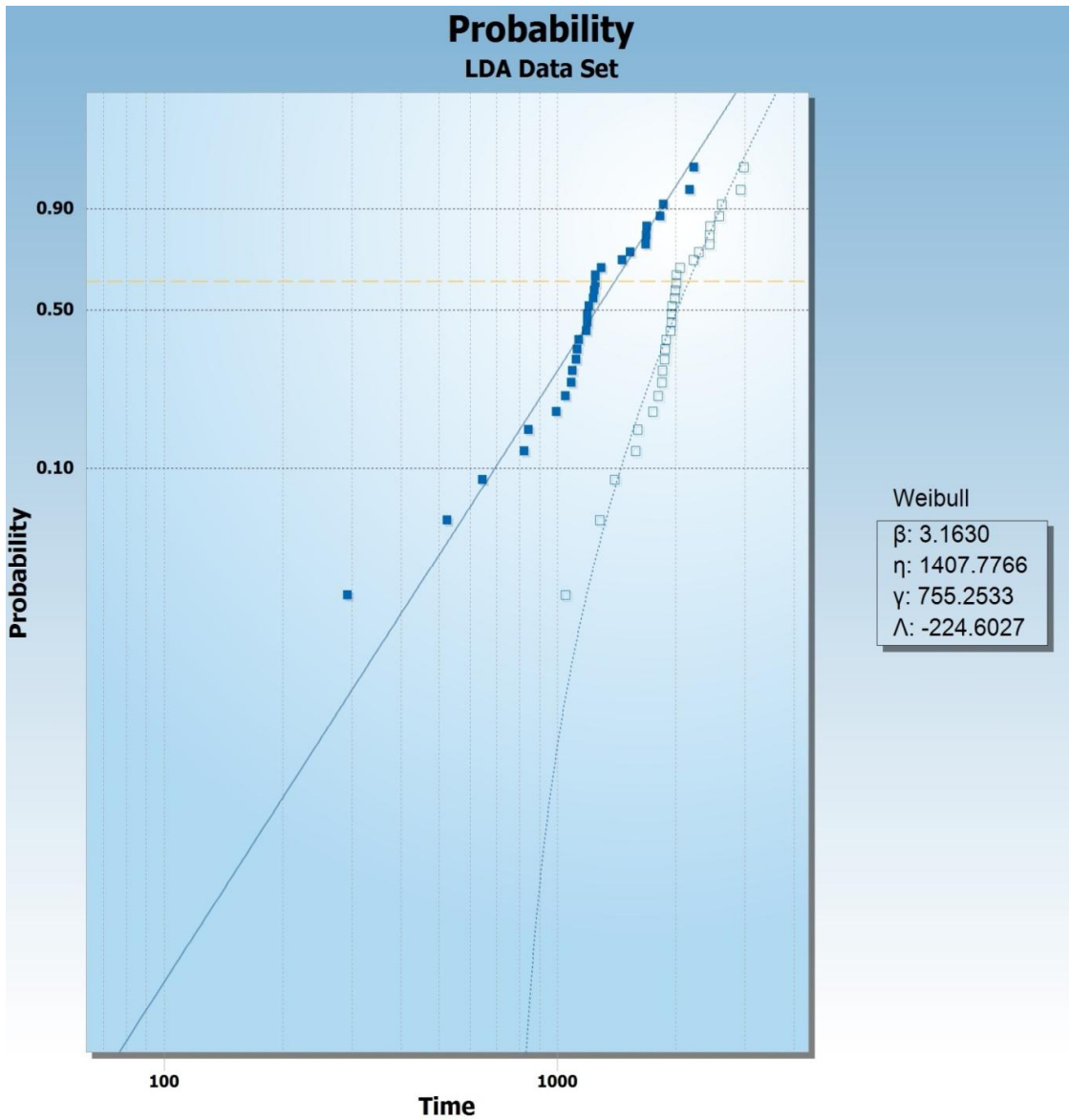


Fig. 1. Probability plot against time

Applying the following results obtained in our analysis,  $t = 1547.86hr$ ,  $\eta = 1470.7766$ ,  $\beta = 3.163$  and  $\gamma = 755.2533hr$

Using equation (7), the reliability of the three-parameter Weibull distribution is calculated as follows:

$$R(t) = e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}$$

$$R(t) = e^{-\left(\frac{1547.86 - 755.2533}{1470.7766}\right)^{3.163}}$$

$$R(t) = e^{-\left(\frac{792.6067}{1470.7766}\right)^{3.163}}$$

$$R(t) = e^{-0.5389^{3.163}}$$

$$R(t) = e^{-0.1565}$$

$$R(t) = 0.855 \tag{11}$$

From equation (8), the failure rate is calculated as follows:

$$\lambda(t) = \frac{3.163}{14\ 077\ 7\ 66} \left(\frac{154\ 78\ 6-7\ 552\ 533}{14\ 077\ 7\ 66}\right)^{3.163-1}$$

$$\lambda(t) = \frac{3.163}{14\ 077\ 7\ 66} \left( \frac{154\ 78\ 6-7\ 52533}{14\ 077\ 7\ 66} \right)^{2.163}$$

$$\lambda(t) = 0.0022468(0.5389)^{2.163}$$

$$\lambda(t) = 0.0022468 \times 0.2626$$

$$\lambda(t) = 0.00058995 \text{ /hr} \quad (12)$$

$$MTTF = 755.2533 + 1407.7766 \Gamma \left( 1 + \frac{1}{3.163} \right)$$

$$MTTF = 755.2533 + 1407.7766 \Gamma(1.32)$$

From the Gamma Function table,  $\Gamma(1.32) = 0.8946$

$$MTTF = 755.2533 + 1407.7766 \times 0.8946$$

$$MTTF = 755.2533 + 1259.3969$$

$$MTTF = 2014.65hr \quad (13)$$

From equation (9), the Mean Time to Failure (MTTF) of the three-parameter Weibull distribution is calculated:

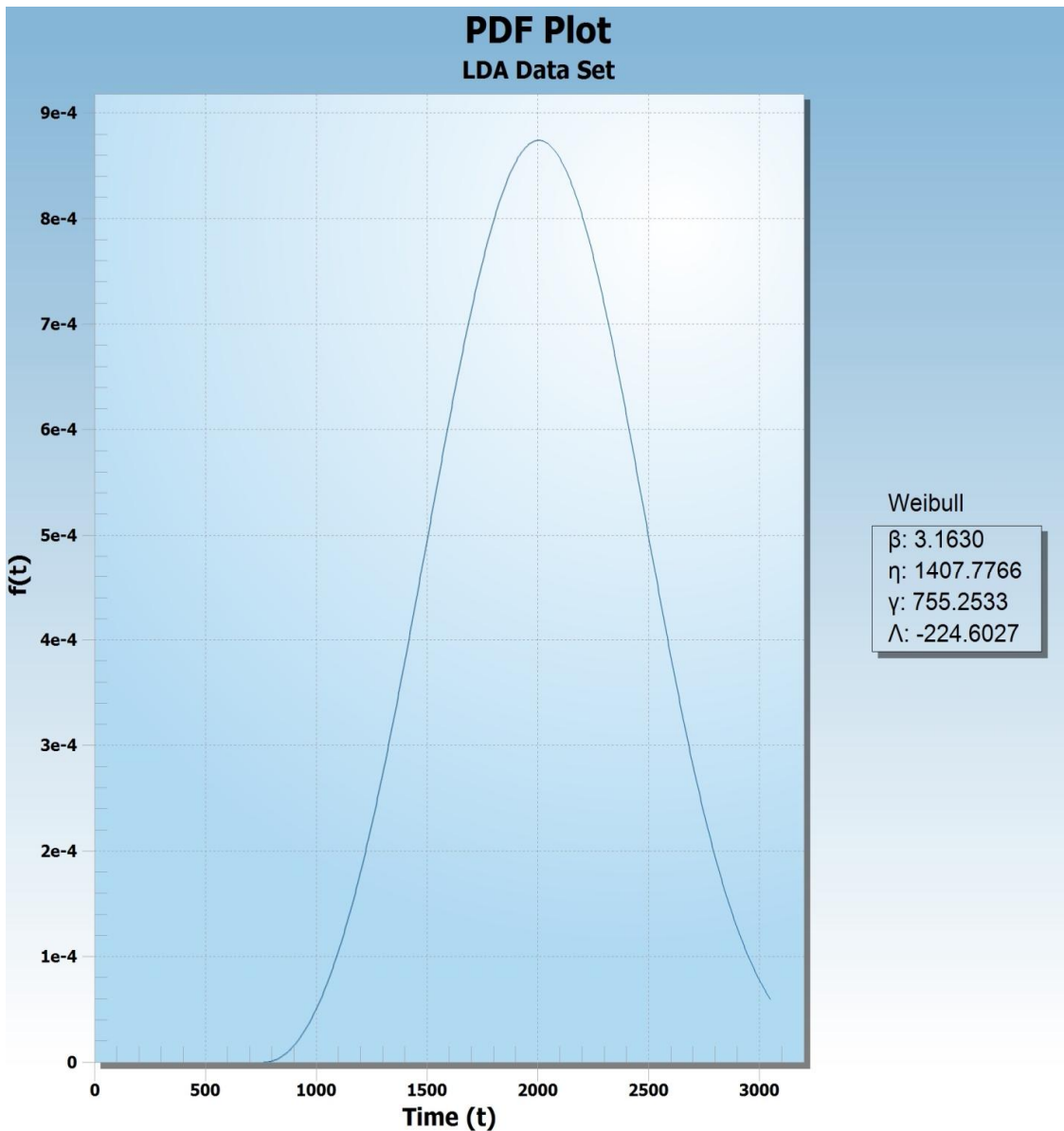


Fig. 2. Pdf plot against time

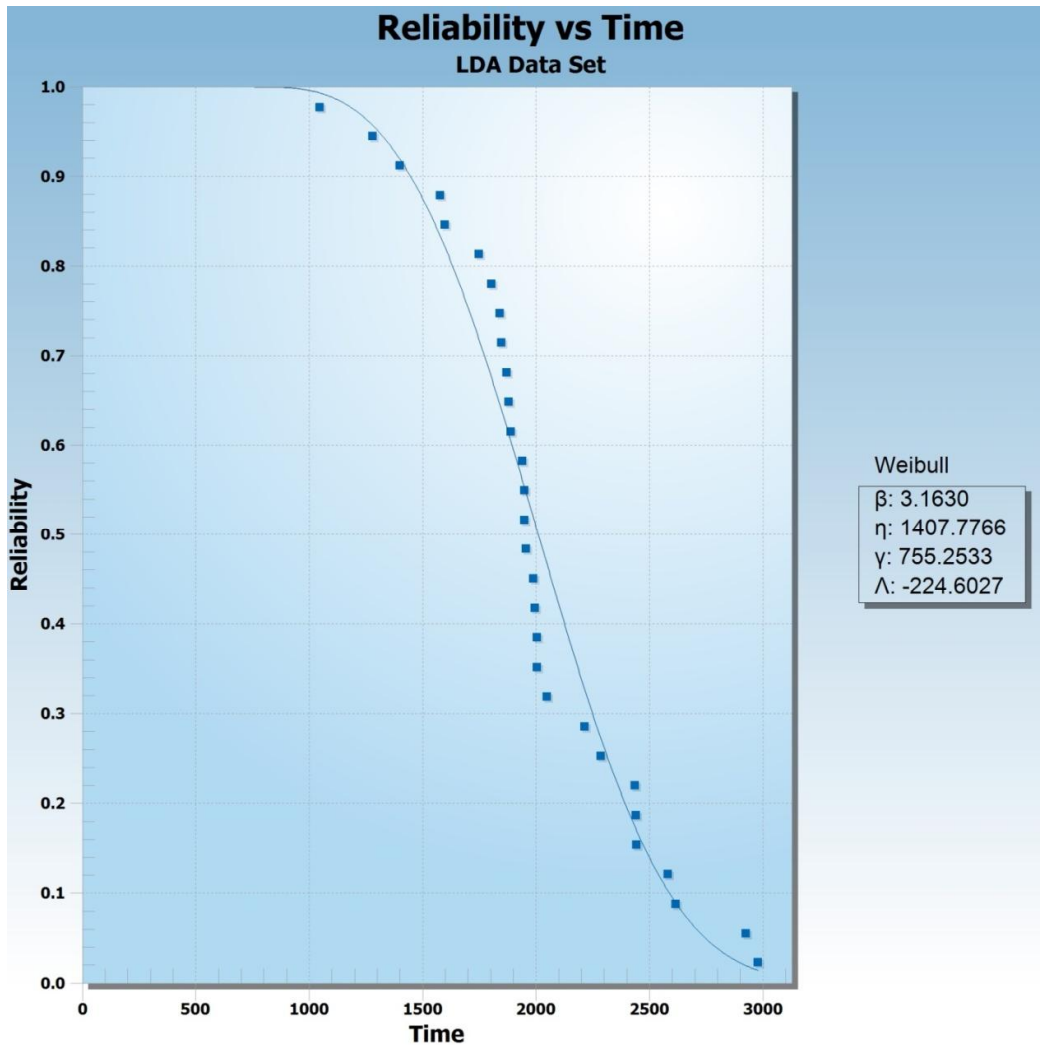


Fig. 3. Reliability plot against time

### 3.2 Discussion

In this study, TPC Windchill Quality Solutions 11 software package is used to analyze the failure times of ten identical gas turbine blades and the following results were obtained for the three-parameter Weibull estimates;  $\gamma = 755hr$ ,  $\beta = 3.163$ ,  $\eta = 1408 hr$ , and time to failure,  $t = 1548 hr$  as shown in Tab 2. The result obtained for the location parameter,  $\gamma = 755hr$ , indicates that the use of the three-parameter Weibull model is appropriate. From literature, when the shape parameter,  $\beta > 1$ , it indicates that the type of failure is the wear-out period or increasing failure rate (IFR). Therefore, since the result for the shape parameter,  $\beta$  is greater than 1, this shows that the gas turbines blades have started to wear having been in operation for a very long

time. This type of failure can be reduced by carrying out preventive maintenance as well as parts replacement technology. In Tab 3, the data used in this study was analyzed using reliability software Minitab 19 to obtain the Weibull estimates and the results compared with those obtained using TPC Windchill Quality Solutions 11. The values of the three-parameter Weibull estimates in both analyses were observed to be very close. In Fig. 1, the probability of failure over time is shown. From the plot, the original (unshifted) probability line is seen curving inward (concave down) while the adjusted (shifted) probability line is observed to be linear. The slope of the probability plot can be seen to be decreasing at the beginning but as it gets to the end of the plot, a gradual increase is observed. The slope of the Weibull probability plot is also

observed to be pushed to the right. This is because the value of the location parameter is positive. Fig. 2 shows the plot of the probability density function. From the plot, the probability density function (PDF) is seen increasing steadily up to a certain time,  $t=2000\text{hr}$ ; but from this point further, a sharp decrease in the probability density function is observed. The plot of reliability against time is shown in Fig. 3. From the plot, we observed that the reliability was initially very high at the beginning, but over time, it can be seen to be decreasing. This shows that as the gas turbine blades are aging, there will be a gradual drop in reliability. Fig. 4 shows the plot of the failure rate against time. From Fig. 4, we observe that the failure rate is increasing as time is increasing. This indicates that the gas turbine blades are in their wear-out period. It was also

observed that the slope of the failure rate curve decreases as the scale parameter increases. Fig. 5 shows the contour plot in 3D. From the plot, we observe that the values of the estimates of the three-parameter Weibull vary along the contour axes. It is observed, that as we move from the bottom of the contour to the top, the values of the location and shape parameters can be seen to be increasing gradually while the value of the scale parameter is observed to be slightly affected. The reliability, failure rate and mean time to failure were successfully computed using equations (9-11) and the results shown in equations (10-13). From the result obtained, the probability that the gas turbine blades will continue to operate properly without failure is 85.5% while the mean time to failure of the gas turbine blades is 2,015hrs.

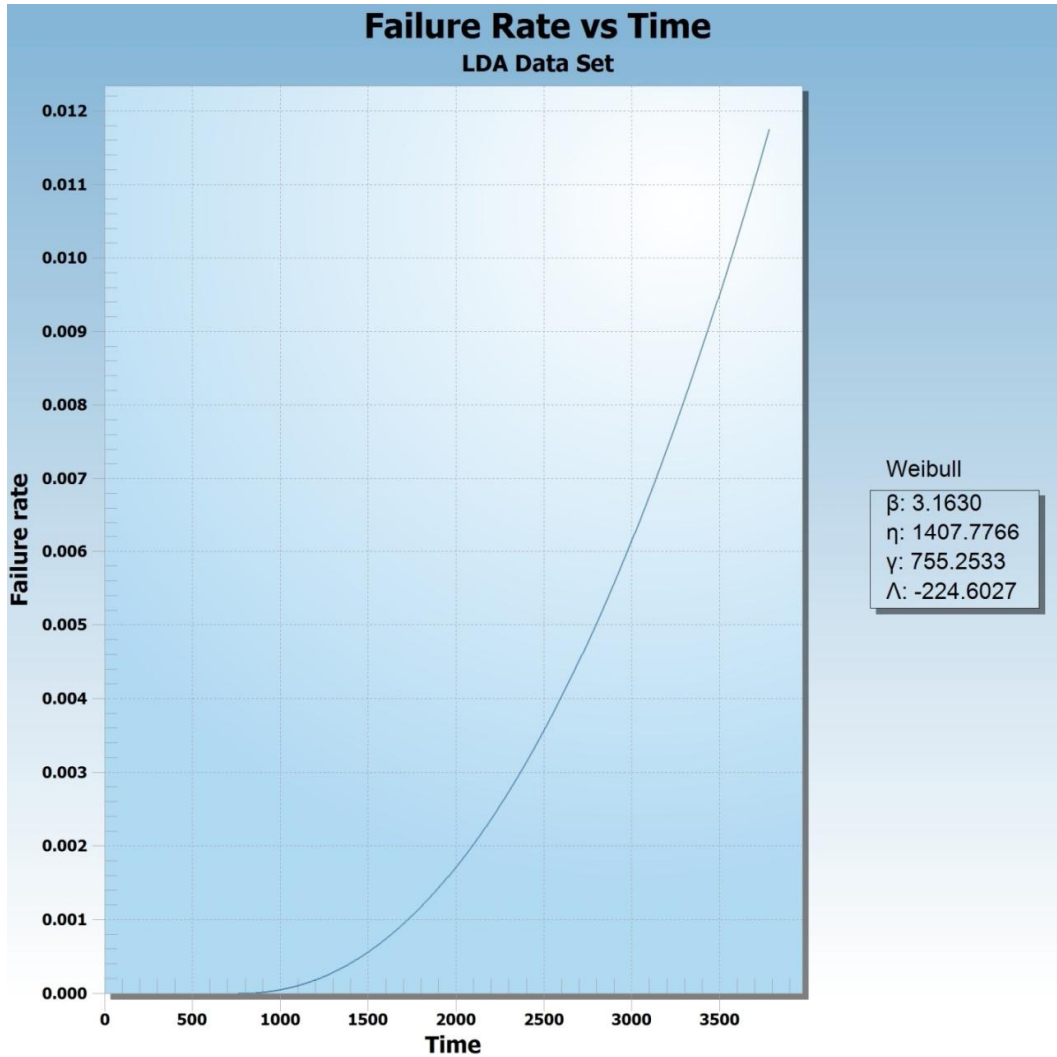


Fig. 4. Failure rate plot against time



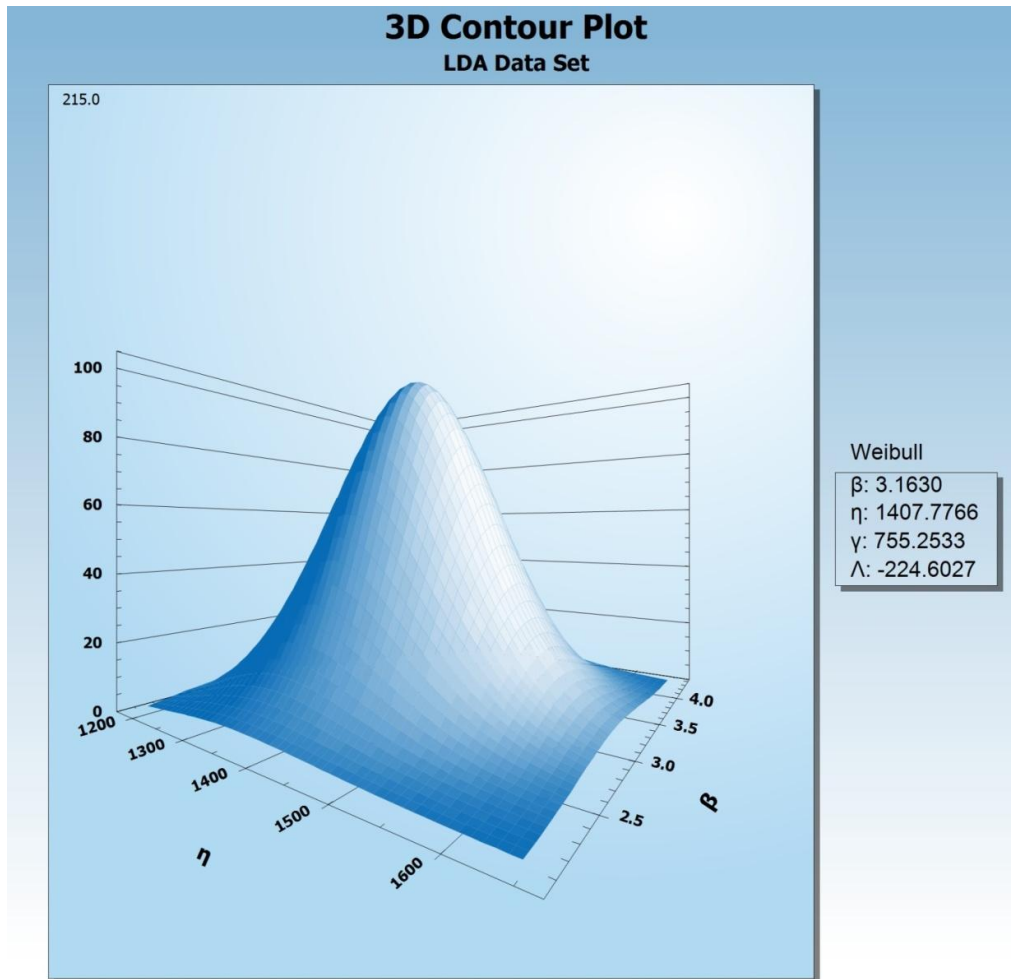


Fig. 5. 3D Contour plot

#### 4. CONCLUSION

The parameter estimation of three-parameter Weibull distribution using maximum likelihood method has been studied. The study involved the analysis of failure times of ten identical gas turbine blades using TPC Windchill Quality Solutions 11 software package to obtain the estimates of the three-parameter Weibull, namely, the location, shape and scale parameters. The estimates were successfully obtained and the results compared with those obtained using Weibull analysis software, Minitab 19, and the values of the Weibull estimates in both analyses were found to be very close. The results of the analysis indicated that the blades were already in their wear-out period as the value of the shape parameter obtained is greater than 1. From the Weibull estimates obtained, the reliability, failure rate and mean time to failure of

the blades were computed. In this study, consideration was done for maximum likelihood method to successfully obtain three-parameter Weibull estimates of gas turbine blades.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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