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Radiative Effects on Unsteady MHD Couette Flow through a Parallel Plate with Constant Pressure Gradient

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Authors' contributions

This work was carried out in collaboration among all authors. Author ROO designed the study. Author EOA performed the model analysis, wrote the protocol and wrote the first draft of the manuscript. Authors MDS and AL managed the analyses of the study. Author EOA managed the literature searches. All authors read and approved the final manuscript.

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Abstract

In this paper, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plates while the fluid motion is subjected to the constant pressure gradient. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using the Eigenfunction expansion technique. The influence of some emerging non-dimensional parameters namely, pressure gradient, suction parameter, radiation parameter, and Hartman number are examined in detail. It is observed that the primary velocity is increased with increasing pressure gradient while the increase in radiation parameter leads to adecrease in the thermal profile of the flow.

Keywords: Eigenfunction expansion technique; magnetohydrodynamics (MHD); constant pressure gradient; suction; hall current.

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1 Introduction

The dynamics of fluids through the porous channel have been a popular area of research regarding to numerous increasing applications in chemical, mechanical, and material process engineering. Examples of such fluid include clay coating, coal, oil slurries, shampoo, paints cosmetic products, grease, custard and physiological liquids (blood, bile, and synovial fluid). Over the years, considerable interest has been observed on the effect of MHD in viscous, incompressible, non-Newtonian fluid flow with heat transfer. These interests on non-Newtonian fluids are owed to its important applications in various branches of science, engineering and technology, particularly in chemical and nuclear industries, material processing, geophysics and bio-engineering. In view of these applications, an extensive range of mathematical models have been developed to simulate the diverse hydrodynamic behavior of these non-Newtonian fluids. However, different non-Newtonian fluid models have been presented by researchers and solved using various types of analytical and computational schemes.

Sayed-Ahmed et al. [1] investigated time dependent pressure gradient effect on unsteady MHD Couette flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel non-conducting porous plates with heat transfer under an exponential decaying pressure gradient. Olayiwola [2] investigated the modeling and simulation of combustion fronts in porous media. Jana et al. [3] examined Couette flow through a porous medium in a rotating system. In another related work, Seth et al. [4] studied the effects of rotation and magnetic field on unsteady Couette flow in a porous channel. Seth et al. [5] studied the unsteady hydromagnetic Couette flow within porous plates in a rotating system. Recently, Sharma and Yadav [6] considered Heat transfer through three dimensional Couette flow between a stationary porous plate bounded by porous medium and moving porous plates. Sharma et al. [7] investigated the steady laminar flow and heat transfer of a non-Newtonian fluid through a straight horizontal porous channel in the presence of a heat source. Olayiwola and Ayeni [8] examined a mathematical model and simulation of In-situ combustion in porous media. In another development, a mathematical model of solid fuel Arrhenius combustion in a fixed-bed was analyzed by Olayiwola [9]. Bhattacharyya et al. [10] studied analytically the solution for magnetohydrodynamic boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer. The unsteady boundary layer flow of a Casson fluid due to an impulsively started moving plate was considered by Mustafa et al. [11]. Recently, Mukhopadhyay et al. [12] investigated the steady boundary layer flow and heat transfer over a porous moving plate in the presence of thermal radiation. Makinde and Mhone [13] studied the heat transfer to MHD flow in a channel filled with a porous medium. Malapati and Polarapu [14] analyzed unsteady MHD free convective heat and mass transfer in a boundary layer flow past a vertical permeable plate with thermal radiation and chemical reaction. Chamkha and Ahmed [15] examined unsteady MHD heat and mass transfer by mixed convection flow in the forward stagnation region of a rotating sphere at different wall conditions. The effects of thermal radiation and magnetic field on unsteady mixed convection flow and heat transfer over a stretching in the presence of internal heat generation/absorption was studied by Elbashbeshy and Aldawody [16]. Talukdar [17] investigated the buoyancy and chemical reaction effects on MHD mixed convection heat and mass transfer in a porous medium with thermal radiation and ohmic heating. Mohammed et al. [18] analyzed radiation and mass transfer effects on MHD oscillatory flow in a channel filled with porous medium in the presence of chemical reaction.

The aim of the research is to establish an analytical solution capable of describing the concentration, temperature and velocity in the process of MHD Couette flow through a parallel plate with constant pressure gradient.

2 Mathematical Formulation

Following Sayed-Ahmed et al. [1], the unsteady flow of a viscous, incompressible, non-conducting fluid through a channel with chemical reaction, thermal radiation, constant and variable pressure gradient in the presence of magnetic field is investigated. The flow is assumed to be laminar, incompressible and flows between two infinite horizontal plates located at $y = \pm h$ which extends from $x = -\infty$ to ∞ and from $z = -\infty$ to ∞.

The upper plate is suddenly set into motion and moves with a uniform velocity U_0 while the lower plate is kept stationary as shown in the diagram below. The upper plate is simultaneously subjected to a step change in temperature from T_1 to T_2. The upper and lower plates are kept at two constant temperatures T_2 and T_1 respectively with $T_2 > T_1$. The fluid flows between the two plates under the influence of an exponential decaying with time pressure gradient in the x-direction which is a generalization of a constant pressure gradient. A uniform suction from above and injection from below with constant velocity v_0 which are all applied at $t = 0$. The system is subjected to a uniform magnetic field B_0 in the positive y-direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a small magnetic Reynolds number. The Hall effect is taken into consideration and consequently a z-component of the velocity is expected to arise.

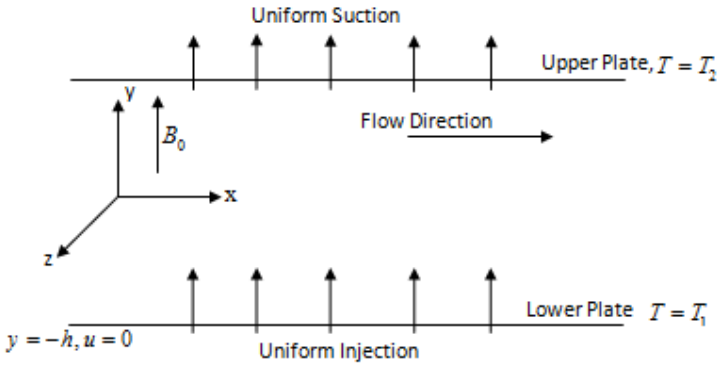

Fig. 1. Schematic diagram of the problem

Based on the above assumptions,

$$v = ui + v_0 j + wk \tag{1}$$

Introducing a Chapman-Rubesin viscosity law, with $w = 1$ as shown in Olayiwola (2016) and using the condition at the lower plate, results in:

$$\mu = \frac{c\mu_1 T}{T_1} \tag{2}$$

Where μ_1 is the Casson coefficient of viscosity.

Thus, the two components of the governing momentum equation in thedimensional form are as follows:

$$\rho\frac{\partial u}{\partial t} + \rho v_0 \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{(1+BiBe)^2 + Be^2}\left[(1+BiBe)u + Bew\right] - \tag{3}$$

$$\mu\frac{u}{k} + g\beta_T\left(T - T_1\right) + g\beta_C\left(C - C_1\right)$$

$$\rho\frac{\partial w}{\partial t}+\rho v_0\frac{\partial w}{\partial y}=\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right)-\frac{\sigma B_0^2}{(1+BiBe)^2+Be^2}\left[(1+BiBe)w-Beu\right]-\mu\frac{w}{k} \tag{4}$$

The energy equation in dimensional form is given as

$$\left.\begin{aligned}\rho C_P\frac{\partial T}{\partial t}+\rho C_P v_0\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right)+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\\ \frac{\sigma B_0^2}{(1+BiBe)^2+Be^2}\left[u^2+w^2\right]-\frac{1}{\rho C_P}\frac{\partial q}{\partial y}\end{aligned}\right\} \tag{5}$$

The concentration equation in dimensional form is given as:

$$\rho\frac{\partial C}{\partial t}+\rho v_0\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial}{\partial y}\left(\mu\frac{\partial C}{\partial y}\right)+D_1\frac{\partial^2 T}{\partial y^2}-D_2\left(C_2-C_1\right) \tag{6}$$

Subject to the initial and boundary conditions;

$$\left.\begin{aligned}&u(y,0)=0, && u(-h,t)=0, && u(h,t)=U_0\\ &w(y,0)=U_0y(1-y), && w(-h,t)=0, && w(h,t)=0\\ &T(y,0)=0, && T(-h,t)=T_1, && T(h,t)=T_2\\ &C(y,0)=0, && C(-h,t)=C_1, && C(h,t)=C_2\end{aligned}\right\} \tag{7}$$

Where ρ and μ are respectively the density and apparent viscosity of the fluid, δ is electric conductivity, β is Hall factor, Bi is ion slip parameter, $Be=\sigma\beta B_0$ is Hall parameter, c and k are the specific heat capacity and thermal conductivity of the fluid respectively. Where u and w are components of velocities along and perpendicular to the plate in x and y directions respectively, β_T is the coefficient of volume expansion of the moving fluid, β_C is the coefficient of volumetric expansion with concentration, v is the kinematic viscosity, T is the temperature of the fluid, C is the concentration of the fluid, C_1 is the concentration at infinity, D_1 the thermal diffusivity, D_2 the chemical reaction rate constant, C_P is the specific heat capacity at constant pressure. t is the time, g is the gravitational force, μ_e is the magnetic permeability of the fluid, K is the porous media permeability coefficient, q is radiative heat flux, H_0 is the intensity of the magnetic field, $B_0=\mu_e H_0$ is electromagnetic induction, τ_0 is yield stress, β is coefficient of volume expansion due to temperature and α is the mean radiation absorption coefficient.

To write the governing dimensional equations (3)-(6) with their corresponding boundary conditions (7) in non-dimensional form, we use the following dimensionless variables:

$$\bar{u}=\frac{u}{U_0},\qquad \bar{w}=\frac{w}{U_0},\qquad \bar{y}=\frac{y}{h},\qquad \bar{x}=\frac{x}{h},\qquad \bar{t}=\frac{tU_0}{h},\qquad \theta=\frac{T-T_1}{T_2-T_1}$$

$$\varphi=\frac{C-C_1}{C_2-C_1},\qquad \bar{P}=\frac{P}{\rho U_0^2},\qquad \bar{\mu}=\frac{c\mu_1 T}{T_1},\qquad \sigma\left(pressure\ gradient\right)=-\frac{\partial \bar{P}}{\partial x} \tag{8}$$

and we obtain

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=\sigma+\frac{c}{\mathrm{Re}}\frac{\partial}{\partial y}\left((a\theta+1)\frac{\partial u}{\partial y}\right)-\frac{Ha^2}{\mathrm{Re}\left(1+BiBe\right)^2+Be^2}\left[\left(1+BiBe\right)u+Bew\right]-$$

$$\frac{Pc}{\mathrm{Re}}\left((a\theta+1)u\right)+Gr_\theta\theta+Gr_\phi\phi \tag{9}$$

$$\frac{\partial w}{\partial t}+S\frac{\partial w}{\partial y}=\frac{c}{\mathrm{Re}}\frac{\partial}{\partial y}\left((a\theta+1)\frac{\partial w}{\partial y}\right)-\frac{Ha^2}{\mathrm{Re}\left(1+BiBe\right)^2+Be^2}\left[\left(1+BiBe\right)w-Beu\right]-$$

$$\frac{Pc}{\mathrm{Re}}\left((a\theta+1)w\right) \tag{10}$$

$$\frac{\partial \theta}{\partial t}+S\frac{\partial \theta}{\partial y}=\frac{c}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial}{\partial y}\left[(a\theta+1)\frac{\partial \theta}{\partial y}\right]+\frac{cEc}{\mathrm{Re}}(a\theta+1)\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+$$

$$\frac{EcHa^2}{\mathrm{Re}\left[(1+BiBe)^2+Be^2\right]}\left[u^2+w^2\right]-Ra^2\theta \tag{11}$$

$$\frac{\partial \phi}{\partial t}+S\frac{\partial \phi}{\partial \bar{y}}=\frac{c}{Sc\,\mathrm{Re}}\frac{\partial}{\partial y}\left[(a\theta+1)\frac{\partial \phi}{\partial y}\right]+T_D\frac{\partial^2\theta}{\partial \bar{y}^2}-Kr\phi \tag{12}$$

Subject to the initial and boundary conditions

$$\left.\begin{array}{lll} u\left(y,0\right)=0, & u\left(-1,t\right)=0, & u\left(1,t\right)=1 \\ w\left(y,0\right)=y\left(1-y\right), & w\left(-1,t\right)=0, & w\left(1,t\right)=0 \\ \theta\left(y,0\right)=0, & \theta\left(-1,t\right)=0, & \theta\left(1,t\right)=1 \\ \phi\left(y,0\right)=0, & \phi\left(-1,t\right)=0, & \phi\left(1,t\right)=1 \end{array}\right\} \tag{13}$$

Where

$$\left.\begin{array}{llll} \mathrm{Re}=\dfrac{\rho U_0 h}{\mu_1}, & S=\dfrac{\nu_0}{U_0}, & P=\dfrac{h^2\mu_1}{k}, & Ha^2=\dfrac{\delta B_0^2 h^2}{\mu_1}, \quad Gr_\theta=\dfrac{g\beta_T\left(T_2-T_1\right)h}{\rho U_0^2}, \\ Gr_\phi=\dfrac{g\beta_C\left(C_2-C_1\right)h}{\rho U_0^2}, & \mathrm{Pr}=\dfrac{\mu_1 c_p}{k}, & Ec=\dfrac{U_0^2}{c_p\left(T_2-T_1\right)}, & Ra^2=\dfrac{4\alpha^2 h}{\rho^2 C_p^2 U_0}, \\ Sc=\dfrac{U_0 h}{D}, & T_D=\dfrac{D\left(T_2-T_1\right)}{h\left(C_2-C_1\right)U_0}, & a=\dfrac{T_2-T_1}{T_1}, & Kr=\dfrac{D_2 h}{\rho U_0}, \end{array}\right\} \quad (14)$$

3 Method of Solution

3.1 Transformation

Since the boundary conditions are from -1 to 1, we first transform the boundary conditions to 0 to 1 using the transformation:

$$z=\frac{y+1}{2} \quad (15)$$

Let $0<S<<1$ and $Ec=bS, \quad a=eS, \quad Be=fS, \quad Gr_\theta=gS, \quad Gr_\phi=hS$ such that

$$\left.\begin{array}{l} u\left(z,t\right)=u_0\left(z,t\right)+Su_1\left(z,t\right)+\ldots \\ w\left(z,t\right)=w_0\left(z,t\right)+Sw_1\left(z,t\right)+\ldots \\ \theta\left(z,t\right)=\theta_0\left(z,t\right)+S\theta_1\left(z,t\right)+\ldots \\ \phi\left(z,t\right)=\phi_0\left(z,t\right)+S\phi_1\left(z,t\right)+\ldots \end{array}\right\} \quad (16)$$

Collecting like powers of S, we have for:

$$\left.\begin{array}{l} S^0: \\ \dfrac{\partial\theta_0}{\partial t}=\dfrac{c}{4\,\mathrm{Re}\,\mathrm{Pr}}\dfrac{\partial}{\partial z}\left[\dfrac{\partial\theta_0}{\partial z}\right]-Ra^2\theta_0 \\ \\ \theta_0\left(z,0\right)=0, \quad \theta_0\left(0,t\right)=0, \quad \theta_0\left(1,t\right)=1 \end{array}\right\} \quad (17)$$

$$\left.\begin{aligned}&\frac{\partial \phi_0}{\partial t}=\frac{c}{4Sc\,\mathrm{Re}}\frac{\partial}{\partial z}\left[\frac{\partial \phi_0}{\partial z}\right]+T_D\frac{\partial^2\theta_0}{\partial z^2}-Kr\phi_0\\&\phi_0(z,0)=0,\quad \phi_0(0,t)=0,\quad \phi_0(1,t)=1\end{aligned}\right\}\tag{18}$$

$$\left.\begin{aligned}&\frac{\partial w_0}{\partial t}=\frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial w_0}{\partial z}\right)-\frac{Ha^2}{\mathrm{Re}}[w_0]-\frac{Pc}{\mathrm{Re}}[w_0]\\&w_0(z,0)=(2z-1)(2-2z),\quad w_0(0,t)=0,\quad w_0(1,t)=0\end{aligned}\right\}\tag{19}$$

$$\left.\begin{aligned}&\frac{\partial u_0}{\partial t}=\frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{\partial u_0}{\partial z}\right)-\frac{Ha^2}{\mathrm{Re}}[u_0]-\frac{Pc}{\mathrm{Re}}(u_0)+\sigma\\&u_0(z,0)=0,\quad u_0(0,t)=0,\quad u_0(1,t)=1\end{aligned}\right\}\tag{20}$$

S^{1}:

$$\left.\begin{aligned}&\frac{\partial u_1}{\partial t}+\frac{1}{2}\frac{\partial u_0}{\partial z}=\frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(\frac{e\theta_0\partial u_0}{\partial z}+\frac{\partial u_1}{\partial z}\right)-\frac{Ha^2}{\mathrm{Re}}[u_0Bif+u_1+fw_0]-\frac{Pc}{\mathrm{Re}}(e\theta_0u_0+u_1)+\\&g\theta_0+h\phi_0\\&u_1(z,0)=0,\qquad u_1(0,t)=0,\qquad u_1(1,t)=0\end{aligned}\right\}\tag{21}$$

$$\left.\begin{aligned}&\frac{\partial \theta_1}{\partial t}+\frac{1}{2}\frac{\partial \theta_0}{\partial z}=\frac{c}{4\,\mathrm{Re}\,\mathrm{Pr}}\frac{\partial}{\partial y}\left[e\theta_0\frac{\partial \theta_0}{\partial z}+\frac{\partial \theta_1}{\partial z}\right]+\frac{bc}{4\,\mathrm{Re}}\left[\left(\frac{\partial u_0}{\partial y}\right)^2+\left(\frac{\partial w_0}{\partial y}\right)^2\right]-\\&\frac{bHa^2}{\mathrm{Re}}\left[(u_0)^2+(w_0)^2\right]-Ra^2\theta_1\\&\theta_1(z,0)=0,\qquad \theta_1(0,t)=0,\qquad \theta_1(1,t)=0\end{aligned}\right\}\tag{22}$$

$$\left.\begin{aligned} &\frac{\partial w_1}{\partial t}+\frac{1}{2}\frac{\partial w_0}{\partial z}=\frac{c}{4\,\mathrm{Re}}\frac{\partial}{\partial z}\left(e\theta_0\frac{\partial w_0}{\partial z}+\frac{\partial w_1}{\partial z}\right)-\frac{Ha^2}{\mathrm{Re}}\left[w_0 Bif+w_1-fu_0\right]-\frac{Pc}{\mathrm{Re}}\left(e\theta_0 w_0+w_1\right)\\ &w_1(z,0)=0, \qquad w_1(0,t)=0, \qquad w_1(1,t)=0 \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} &\frac{\partial \phi_1}{\partial t}+\frac{1}{2}\frac{\partial \phi_0}{\partial z}=\frac{c}{4Sc\,\mathrm{Re}}\frac{\partial}{\partial z}\left[e\theta_0\frac{\partial \phi_0}{\partial z}+\frac{\partial \phi_1}{\partial z}\right]+T_D\frac{\partial^2\theta_1}{\partial z^2}-Kr\phi_1\\ &\phi_1(z,0)=0, \qquad \phi_1(0,t)=0, \qquad \phi_1(1,t)=0 \end{aligned}\right\} \tag{24}$$

3.2 Eigenfunction expansion technique

Now, consider the problem (see Myint-U and Debnath, (1987))

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}+\alpha u+F(x,t)\\ &u(x,0)=f(x), \qquad u(0,t)=0, \qquad u(L,t)=0 \end{aligned}\right\} \tag{25}$$

For the solution of problem (25), we assume a solution of the form

$$u(x,t)=\sum_{n=1}^{\infty}u_n(t)\sin\frac{n\pi}{L}x \tag{26}$$

Where

$$u_n(t)=\int_0^t e^{\left(\alpha-k\left(\frac{n\pi}{L}\right)^2\right)(t-\tau)}F_n(\tau)d\tau+b_n e^{\left(\alpha-k\left(\frac{n\pi}{L}\right)^2\right)t} \tag{27}$$

$$F_n(t)=\frac{2}{L}\int_0^L F(x,t)\sin\frac{n\pi}{L}xdx \tag{28}$$

$$b_n(t)=\frac{2}{L}\int_0^L F(x)\sin\frac{n\pi}{L}xdx \tag{29}$$

Comparing equation (17) – (24) with the (25) we obtain the solutions to the velocity (primary and secondary), temperature, and concentration distributions as

$$\theta_0(\mathrm{z},\mathrm{t}) = \mathrm{z} + \sum_{n=1}^{\infty} q_1\left(1 - e^{-q_0 t}\right)\sin n\pi z \tag{30}$$

$$\phi_0(\mathrm{z},\mathrm{t}) = z + v_2(\mathrm{z},\mathrm{t}) \tag{31}$$

$$w_0(\mathrm{z},t) = \sum_{n=1}^{\infty} q_{10} e^{-q_{11}t} Sinn\pi z \tag{32}$$

$$u_0(z,t) = z + \sum_{n=1}^{\infty} \frac{q_{12}}{q_{11}}\left(1 - e^{-q_{11}t}\right) Sinn\pi z \tag{33}$$

$$u_1(\mathrm{z},\mathrm{t}) = \sum_{n=1}^{\infty} u_5(t) Sinn\pi z \tag{34}$$

Where

$$u_5(t) = q_{16}\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right)\right) + \sum_{n=1}^{\infty} q_{27} P_4\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right) - te^{-q_{11}t}\right) +$$

$$\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{28} q_1 P_4\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right) - te^{-q_{11}t} - \frac{1}{q_{11} - q_0}\left(e^{-q_0 t} - e^{-q_{11}t}\right) - \frac{1}{q_0}\left(e^{(q_{11} - q_0)t} - e^{-q_{11}t}\right)\right) +$$

$$\sum_{n=1}^{\infty} q_{29} q_1\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right) - \frac{1}{q_{11} - q_0}\left(e^{-q_0 t} - e^{-q_{11}t}\right)\right) +$$

$$q_{26}\left(\begin{array}{l} \sum_{n=1}^{\infty} q_4\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right) - \frac{1}{q_{11} - q_2}\left(e^{-q_2 t} - e^{-q_{11}t}\right)\right) + \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_5 P_1\left(\frac{1}{q_{11}}\left(1 - e^{-q_{11}t}\right) - te^{-q_{11}t} - \frac{1}{q_{11} - q_2}\left(e^{-q_2 t} - e^{-q_{11}t}\right)\right) - \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_5 P_2\left(\frac{1}{q_{11} - q_0}\left(e^{-q_0 t} - e^{-q_{11}t}\right) - \frac{1}{q_{11} - q_2}\left(e^{-q_2 t} - e^{-q_{11}t}\right)\right) \end{array}\right)$$

$$\theta_1(z,t) = \sum_{n=1}^{\infty} u_6(t)\, Sinn\pi z \tag{35}$$

Where

$$u_6(t) = \begin{pmatrix} \sum_{n=1}^{\infty} q_1 q_{30}\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-te^{-q_0 t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_1^2 q_{31}\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-2te^{-q_0 t}-\frac{1}{q_0}\left(e^{-2q_0 t}-e^{-q_0 t}\right)\right)+ \\ q_{32}\frac{1}{q_0}\left(1-e^{-q_0 t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{10}^2 q_{33}\left(\frac{1}{q_0-2q_{11}}\left(e^{-2q_{11}t}-e^{-q_0 t}\right)\right)+ \\ \sum_{n=1}^{\infty} q_{34}P_4\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-\frac{1}{q_0-q_{11}}\left(e^{-q_{11}t}-e^{-q_0 t}\right)\right)+ \\ \sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{35}P_4^2\left(\frac{1}{q_0}\left(1-e^{-q_0 t}\right)-\frac{2}{q_0-q_{11}}\left(e^{-q_{11}t}-e^{-q_0 t}\right)+\frac{1}{q_0-2q_{11}}\left(e^{-2q_{11}t}-e^{-q_0 t}\right)\right) \end{pmatrix}$$

$$w_1(z,t) = \sum_{n=1}^{\infty} u_7(t)\, Sinn\pi z \tag{36}$$

Where

$$u_7(t) = \begin{pmatrix} q_{10}q_{36}\left(te^{-q_{11}t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty} q_{37}q_1 q_{10}\left(te^{-q_{11}t}+\frac{1}{q_0}\left(e^{-(q_{11}-q_0)t}-e^{-q_{11}t}\right)\right)+q_{38}\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right)\right)+ \\ \sum_{n=1}^{\infty} q_{14}P_4\left(\frac{1}{q_{11}}\left(1-e^{-q_{11}t}\right)+te^{-q_{11}t}\right) \end{pmatrix}$$

$$\phi_1(z,t) = \sum_{n=1}^{\infty} u_8(t)\, Sinn\pi z \tag{37}$$

Where

$$
u_8(t)=\left(\begin{array}{l}
\left(\begin{array}{l}
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{43}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{44}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}- \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{45}\frac{t\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{46}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{47}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}- \\
2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{48}\frac{t\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{49}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{50}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+ \\
q_{39}\left(\begin{array}{l}
\sum_{n=1}^{\infty}q_{51}\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{52}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{53}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-q_0}+ \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{54}\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{55}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{56}\frac{\left(e^{-q_{11}t}-e^{-q_2t}\right)}{q_2-q_{11}}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{57}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+ \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{58}\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}\right)-2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{59}\frac{\left(e^{-q_{11}t}-e^{-q_2t}\right)}{q_2-q_{11}}+2\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{60}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}+ \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{61}\frac{\left(e^{-2q_{11}t}-e^{-q_2t}\right)}{q_2-2q_{11}}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_{62}\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}
\end{array}\right)
\end{array}\right)+ \\
q_{40}\frac{\left(1-e^{-q_2t}\right)}{q_2}+ \\
q_{41}\left(\begin{array}{l}
-\sum_{n=1}^{\infty}q_4\frac{\left(1-e^{-q_2t}\right)}{q_2}+\sum_{n=1}^{\infty}q_4te^{-q_2t}+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_1\frac{\left(1-e^{-q_2t}\right)}{q_2}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_1te^{-q_2t}-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_2\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0} \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5P_2te^{-q_2t}
\end{array}\right) \\
q_{42}\left(\begin{array}{l}
-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_1q_4\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-te^{-q_2t}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_1\left(\frac{\left(1-e^{-q_2t}\right)}{q_2}-te^{-q_2t}\right)-\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_2\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-te^{-q_2t}\right)+ \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_1q_4\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)+\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_1\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)+ \\
\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}q_5q_1P_2\left(\frac{\left(e^{-q_0t}-e^{-q_2t}\right)}{q_2-q_0}-\frac{\left(e^{-(q_2+q_0)t}-e^{-q_2t}\right)}{q_0}\right)
\end{array}\right)
$$

Therefore the solutions to the governing equations are given as:

$$\theta(\mathrm{z},\mathrm{t})=\mathrm{z}+\sum_{n=1}^{\infty}q_1\left(1-e^{-q_0t}\right)\sin n\pi z+\sum_{n=1}^{\infty}u_6(t)\,Sinn\pi z \tag{38}$$

$$\phi(z,t) = z + v_2(z,t) + S\sum_{n=1}^{\infty} u_8(t)\,Sinn\pi z \tag{39}$$

$$w(z,t) = \sum_{n=1}^{\infty} q_{10} e^{-q_{11}t}\,Sinn\pi z + S\sum_{n=1}^{\infty} u_7(t)\,Sinn\pi z \tag{40}$$

$$u(z,t) = z + \sum_{n=1}^{\infty} \frac{q_{12}}{q_{11}}\left(1 - e^{-q_{11}t}\right) Sinn\pi z + S\sum_{n=1}^{\infty} u_5(t)\,Sinn\pi z \tag{41}$$

4 Results and Discussion

The system of partial differential equations describing unsteady couette flow of an electrically conducting incompressible fluid bounded by two parallel non conducting porous plates is solved analytically using eigenfunction expansion method. The analytical solutions of the governing equations are computed and presented graphically with the aid of a computer symbolic algebraic package MAPLE 17 for the values of the following parameters:

$$\mathrm{Re}=1,\quad Ra^2=1,\quad S=0.1,\quad \mathrm{Pr}=0.71,\quad Ha^2=1,\quad Kr=0.5,\quad Sc=0.22,$$

$$Bi=1,\quad Be=1,\quad a=0.1,\quad c=0.2,\quad P=1,\quad T_D=0,\quad Ec=0.01,$$

$$\mathrm{Gr}_\theta=0.2,\quad Gr_\phi=0.2,\quad \sigma=2$$

The Figs. 2-12 Explains the graphs of primary and secondary velocities, temperature and concentration against different dimensionless parameters.

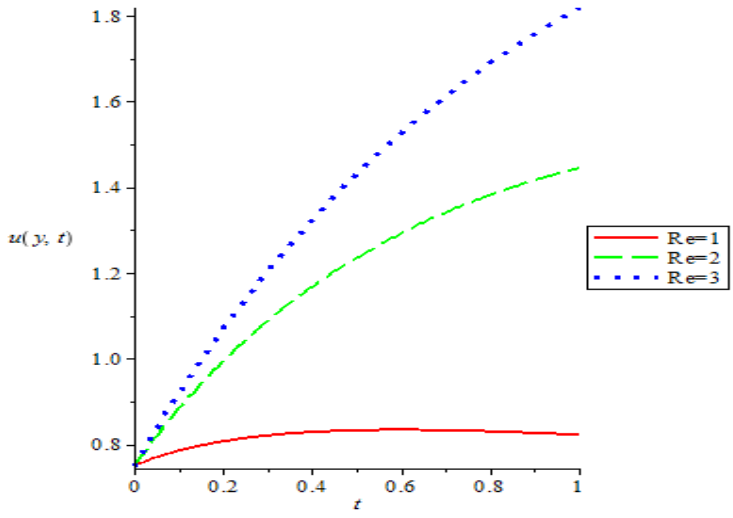

Fig. 2. Relationship between primary velocity and time for different values of re

Fig. 2 presents the graph of primary velocity with time for different values Reynolds number (Re). It is observed that primary velocity increases with time and also increases as Reynolds number increases.

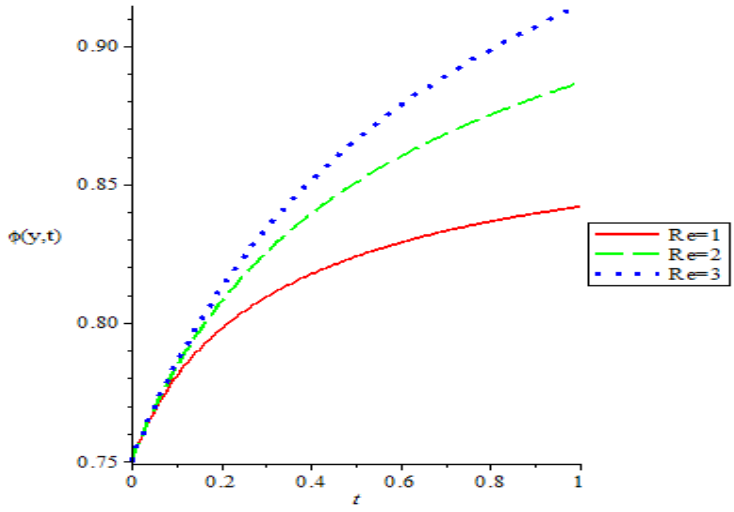

Fig. 3. Relationship between concentration and time for different values of Re

Fig. 3 presents the graph of concentration profile with time t for different values of Reynolds number. It is observed that the concentration profile increases with time and also, increases as Reynolds number increases.

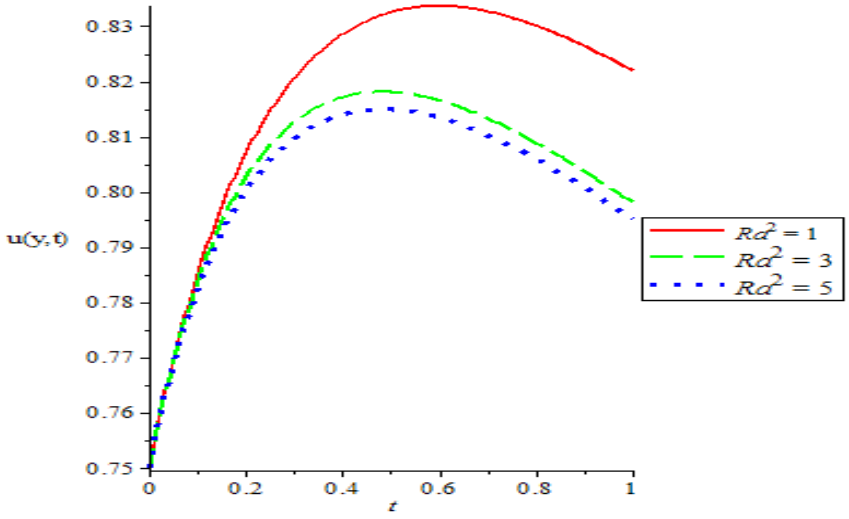

Fig. 4. Relationship between primary velocity and time for different values of Ra

Fig. 4 shows the influence of radiation on the primary velocity profile. It is evident that the primary velocity increases with time. Also, an increase in the radiation parameter is found to decelerate the primary velocity of the flow.

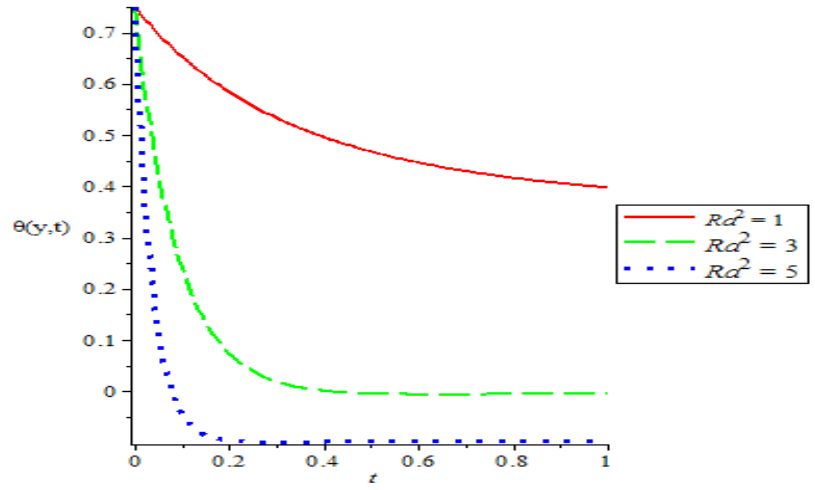

Fig. 5. Relationship between temperature and time for different values of Ra^2

Fig. 5 displays the effect of the thermal radiation parameter on the thermal profile of the flow with time t. It is observed that the flow field suffers a decrease in temperature as radiation parameter increases while as radiation parameter increases the temperature decreases with time t.

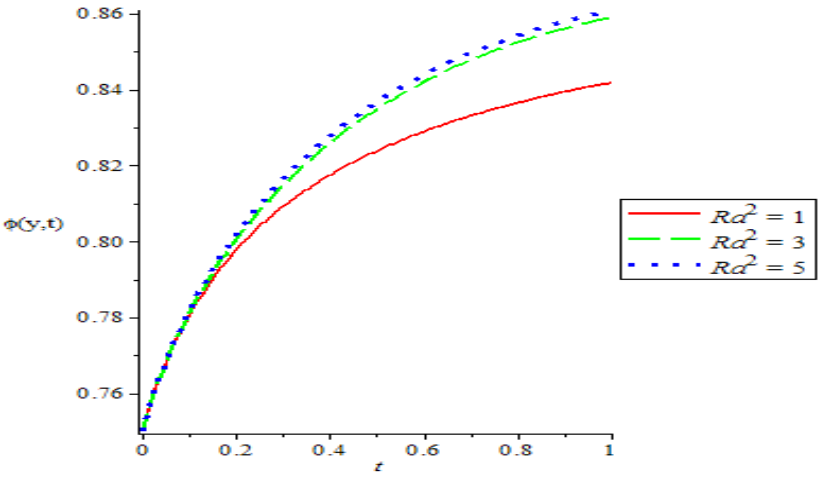

Fig. 6. Relationship between concentration and time for different values of Ra^2

Fig. 6 depicts the graph of concentration with time t for different values of radiation parameter. It is evident that concentration increases with time and also increases as radiation increases.

Fig. 7 illustrates the graph of temperature with time for different values of suction parameter. It is seen that temperature decreases with time and also decreases as suction parameter increases.

Fig. 8 presents the effect of the suction parameter on concentration along distance y. It is observed that an increase in suction parameter leads to decrease in concentration while concentration is observed to increase along distance y.

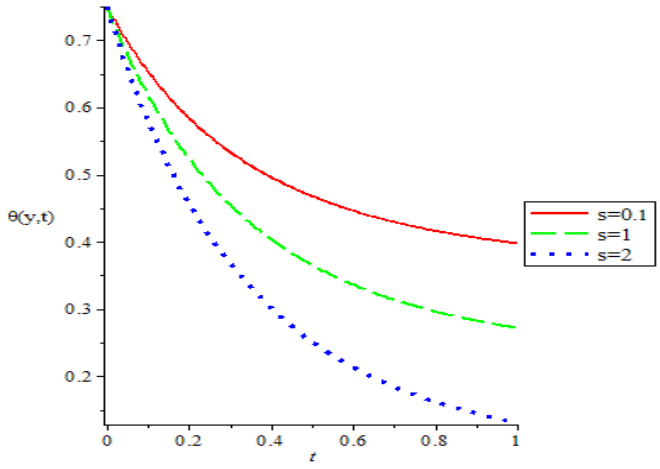

Fig. 7. Relationship between temperature and time for different values of S

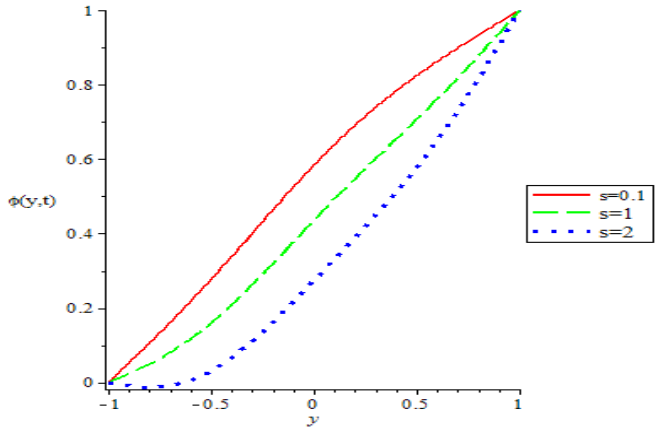

Fig. 8. Relationship between concentration and distance for different values of S

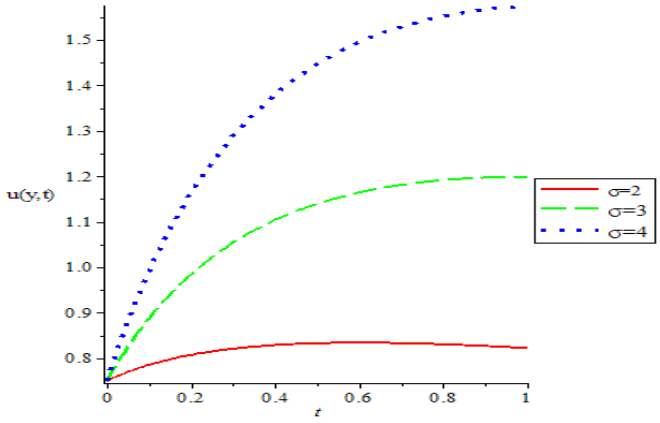

Fig. 9. Relationship between primary velocity and time for different values of σ

Fig. 9 shows the influence of pressure gradient on the primary velocity with time. It is observed that an increase in pressure gradient leads to an increase in primary velocity. Also, primary velocity is found to increase with time.

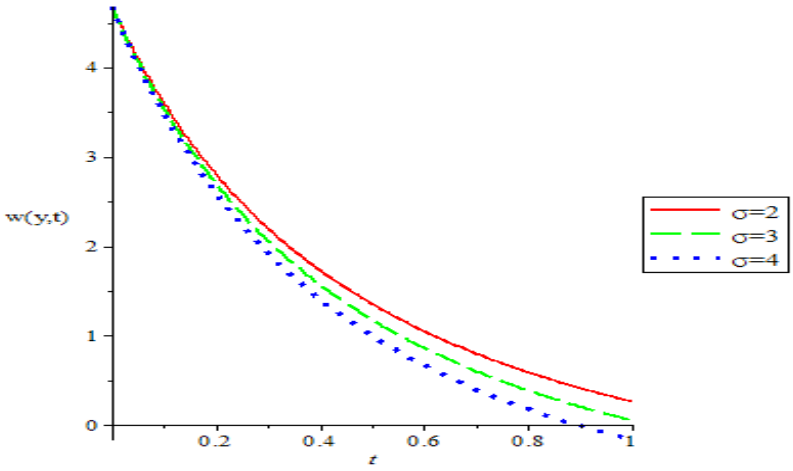

Fig. 10. Relationship between secondary velocity and time for different values of σ

Fig. 10 shows the effect of pressure gradient on secondary velocity with time t. It is observed that increase in pressure gradient leads to decrease in secondary velocity of the fluid while the secondary velocity is observed to decrease with time t.

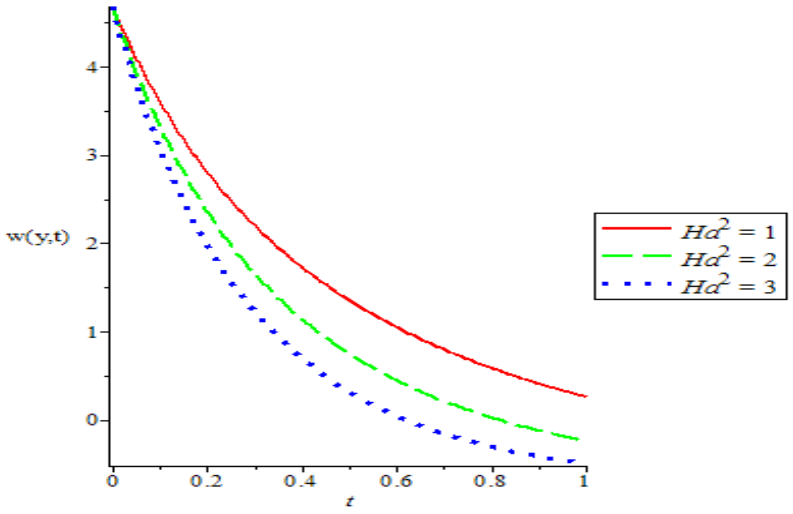

Fig. 11. Relationship between secondary velocity and time for different values of Ha^2

Fig. 11 presents the graph of secondary velocity with time t for different values of Hartman number. It is observed that increase in Hartman number leads to decrease in secondary velocity. This is due to the retarding Lorentz force which acts in opposite direction of the fluid flow when magnetic field is applied. This type of resisting force, slows down the velocity as shown in the figure.

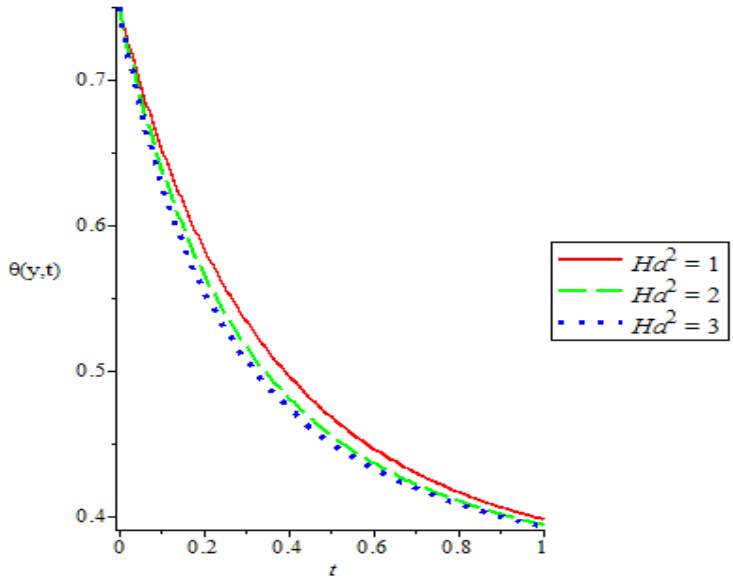

Fig. 12. Relationship between temperature and time for different values of Ha^2

Fig. 12 shows the effect of Hartman number with time t on the temperature profile. It is observed that increase in Hartman number leads to a decrease in temperature. Also, the temperature profile is observed to decrease with time.

5 Conclusion

For constant pressure gradient, the unsteady MHD Couette flow through a porous medium of a viscous incompressible fluid bounded by two parallel porous plates under the influence of thermal radiation and chemical reaction is investigated. A uniform suction and injection are applied perpendicular to the plate. The transformed conservation equations are solved analytically subject to physically appropriate boundary conditions by using the Eigenfunction expansion technique. From the results obtained, we can conclude that:

1. The Increase in Hartman number leads to decrease in velocity. This is due to the retarding Lorentz force which acts in opposite direction of the fluid flow when magnetic field is applied.
2. Concentration profile increases with time and also, increases as Reynolds number increases.
3. The increase in the radiation parameter is found to decelerate the velocity of the flow.
4. The flow field suffers a decrease in temperature as the radiation parameter increases while as radiation parameter the temperature profile is observed to decreases with time t.

Competing Interests

Authors have declared that no competing interests exist.

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