Application of Navier – Stokes Equation to Solve Fluid Flow Problems

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors designed, analyzed and interpreted and prepared the manuscript.

Original Research Article

Abstract

The Navier – Stokes equations were used to obtain the velocity profile for two different fluid flow problems, firstly to a laminar flow through a pipe and secondly to flow of incompressible fluid between two boundaries, one boundary is the air and the other boundary moving with a velocity, inclined at an angleθ. The velocity profiles were obtained and presented in a diagram of showing how the fluid flow through the channels.

1 Introduction

The origin of Navier- stokes equations (NSE) begins with the 1822 paper of C.L.M.H Navier (Ann. Chim.phys. 19, 234-245) who derived equations for homogenous in compressible fluids from a molecular points of view. The continuous derivation of the (NSE) is due to J.C. Saint- Venant (1843) and G.G. stokes (Trans.cambridge philos.soc.1845, 8, 287-319). The (NSE) arc generally treated as the universal basis of

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fluid mechanics, no matter how complicated and unpredictable the behavior of its solutions may be. The (NSE) are the fundamental partial differential equations that describe the flow of fluids, also the motion of viscous fluid substances, such as liquids and gas, with the application of Newton's second law to the fluid motion together with the assumption that the fluid stress is the sum of a diffusion viscous term (proportional to the gradient of velocity) plus pressure term. Many research papers and books has been devoted to the applications of Navier- stokes equations of various degree. Among others see the literatures: Adanhoume [1], studied analytical solution for Navier-stokes equations in the cylindrical coordinates. Adigun [2], gives solution of hydro magnetic flow and heat, transfer past on exponentially stretching permeable vertical heating effect. Haoxiang [3], considered the contra variant form of the (NSE) in time dependent curvilinear coordinate systems. Kaurangini [4], generalized couchette flow in a composite parallel plates channel. Kyle [5], use a volume of fluid method for simulating fluid/fluid interfaces in contact with solid boundaries. Peyret [6], generalized mathematical model for the aqueous humors flow driven by temperature gradient. Sonian[7], study computational methods for fluids flows. Mbah [8], use equation of motion for viscous fluids. M. Veera and Krishna [9], discussed hall effects on MHD peristaltic flow of Jeffrey fluid through porous medium in a vertical stratum. the effects of heat and mass transfer on free convective flow of micro polar fluid were studied over an infinite porous plate in the presence of an inclined magnetic field with a constant suction velocity and taking hall current in to account have been discussed by Veera khirsna etal. krishna and Gangadhar Reddy [10], discussed the unsteady MHD free convection in a boundary layer flow of an electrically conducting fluid through porous medium subject to uniform transverse magnetic field over a moving infinite vertical plate in the presence of heat source and chemical reaction. krishna and Subba reddy [11], have investigated the stimulation on the MHD forced convective flow though stumpy permeable porous medium (oil sands, sand) using lattice Boltzmann method. Krishna and Iyothi [12], discussed the hall effects on MHD rotating flow of a viscous-elastic fluid through a porous medium over an infinite oscillating porous plate with heat source and chemical reaction. Reddy [13], investigated MHD flow of viscous incompressible Nano-fluid through a saturating porous medium. Recently, khrisna et al. [14,15] discuss the MHD flows of an incompressible and electricity conducting fluid in planner channel. Veera khrishna et.al [16], discuss heat and mass transfer on unsteady MHD oscillatory flow of blood through porous arteriole. The effects of radiation and hall current on an unsteady MHD free convective flow in a vertical channel filled with a porous medium have been studied by Veera Krishna [17]. The heat generation/absorption and thermos diffusion on an unsteady free convective MHD flow of radiating and chemically reactive second grade fluid near an infinite vertical plate through a porous medium between two vertical plates under the influence of fluctuating heat source/sink, and chemical reaction. Veera khrisna et al. [18] investigated the heat and mass transfer on MHD free convective flow over an infinite non-conducting vertical flat porous plate. Veera and Iyothi [19] discussed the effect of heat and mass transfer on free convective rotating flow of a visco-elastic incompressible electricity conducting fluid past a vertical porous plate with time independent oscillatory permeability and suction in presence of a uniform transverse magnetic field and heat source. Veera and krishna and Subby reddy [20,21] investigated the transient MHD flow of a reactive second grade fluid through a porous medium between two infinitely long horizontal parallel plates. Veera and krishna et al. [22,23] discussed heat and mass transfer effects on a unsteady flow of a chemically reacting micro polar fluid over an infinite vertical porous plate in the presence of an inclined magnetic field, hall current effect, and thermal radiation taken in to account. Veera Krishna et al. [24]discussed hall effects on steady hydro magnetic flow of a couple stress fluid through a composite medium in a rotating parallel plate channel with porous bed on the lower half. Veera Krishna et al [25], discussed hall effects on unsteady hydro magnetic natural convective rotating flow of second grade fluid past an impulsively moving vertical plate entrenched in a fluid inundated porous medium, while temperature of the plate has temporarily ramped profile. Veera Krishna and Chamkha [26-27], discussed the MHD squeezing flow of a water-based nano fluid through a saturated porous medium between two parallel disks, taking the hall current in to account.veera and Krishna et al. [28]discussed hall effects on MHD peristaltic flow of Jeffrey fluid through porous medium in a vertical stratum. the effects of heat and mass transfer on free convective flow of micropolar fluid were studied over an infinite vertical porous plate in the presence of an inclined magnetic field with a constant suction velocity and taking hall current in to account have been discussed by Veera and Krishna et al. [29] Veera and Krishna and Chamka [30] have discussed the systematic solution of time-dependent mean velocity on MHD peristaltic rotating flow of an electricity conducting couple stress fluid in a uniform elastic porous channel. Yann,T.D.,kunal,p.,jan,N.,VIKtor,l.,such D.,& Steven,H.F (2017,june 3). [31]A simple and efficient

incompressible navier –stokes solver for unsteady complex geometry flows on truncated Domain.journal of computer and fluids,84-94.

The goal of this paper is to show how the Navier – Stokes equations are used to obtain the velocity profile for two different fluid flow problems.

1.1 Preliminary results

The following steps were applied to solve the flow problem:

    STEP 1.   Select a coordinate system for the flow problem.
    STEP 2.   Determine the force(s) that cause the flow.
    STEP 3.   Determine the boundary conditions for the flow problem.
    STEP 4.   Apply the Navier – Stokes equation and then simplify the equations based on the assumption made in Step 1 to 3 above.
    STEP 5.   Solve the resulting equation to obtain the velocity equation (velocity profile) for the flow.

2 Analysis of Results

Problem 1 (Laminar Flow through a pipe):

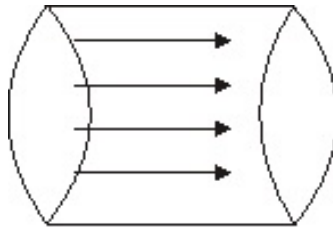Consider a steady laminar flow through a circular pipe with radius, a.

Fig. 1. Laminar flow through a circular pipe

Assumptions:

1. Assume that the flow is in a steady state.
2. Assume that the flow moves in a direction parallel to the pipe only.
3. Assume that there are no movement at the boundary (no slip).

Now applying the steps, we have

STEP 1:

The cylindrical coordinate system is chosen because the pipe is cylindrically symmetric. Thus, we let:

y – axis be parallel to the pipe
x – axis be perpendicular to the y – axis
L = length of the pipe
r = radius of the pipe
v_x = velocity of the flow in the x direction

y_y = velocity of the flow in the y direction
v_θ = velocity of the flow in the θ direction
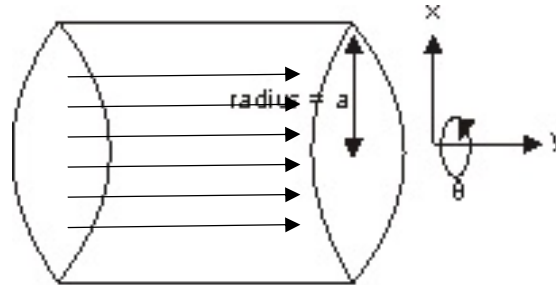
See Fig. 2

Fig. 2. Circular pipe flow with radius a

STEP 2:

Let the pressure at one end be greater than the pressure at the other end with respect to the flow direction.

STEP 3:

The following are the boundary conditions:

At $r = x = 0$, v_y is finite, and
At $r = x = a$, $v_y = 0$ (no slip condition)

STEP 4:

The Navier – Stokes equations (Conservation of momentum) for cylindrical coordinate system are as follows, according to [11]

For the x – axis

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + \frac{v_\theta}{x}\frac{\partial v_x}{\partial \theta} - \frac{v_\theta^2}{r} + v_y \frac{\partial v_x}{\partial y}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left[\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial(x v_x)}{\partial x}\right) + \frac{1}{x^2}\frac{\partial^2 v_x}{\partial \theta^2} - \frac{2}{x^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_x}{\partial y^2}\right] \tag{1}$$

For the y – axis

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + \frac{v_\theta}{x}\frac{\partial v_y}{\partial \theta} - \frac{v_\theta^2}{r} + v_y \frac{\partial v_y}{\partial y}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left[\frac{1}{x}\frac{\partial}{\partial x}\left(x\frac{\partial v_y}{\partial x}\right) + \frac{1}{x^2}\frac{\partial^2 v_y}{\partial \theta^2} + \frac{\partial^2 v_y}{\partial y^2}\right] \tag{2}$$

For the θ – axis

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_x \frac{\partial v_\theta}{\partial x} + \frac{v_\theta}{x}\frac{\partial v_\theta}{\partial \theta} + \frac{v_x v_\theta}{x} + v_y \frac{\partial v_\theta}{\partial y}\right) = -\frac{1}{x}\frac{\partial P}{\partial \theta} + \rho g_\theta + \mu\left[\frac{\partial}{\partial x}\left(\frac{1}{x}\frac{\partial(x v_\theta)}{\partial x}\right) + \frac{1}{x^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{x^2}\frac{\partial v_x}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial y^2}\right] \tag{3}$$

Where

    ρ = density of fluid (assumed constant)
    μ = viscosity of fluid (assumed constant)
    g_x = gravity acting on the fluid in the x direction
    g_y = gravity acting on the fluid in the y direction
    g_θ = gravity acting on the fluid in the θ direction

STEP 4:

At steady state

$$\frac{\partial v_x}{\partial t} = \frac{\partial v_y}{\partial t} = \frac{\partial v_\theta}{\partial t} = 0$$

and since the flow depends on pressure

$$g_x = g_y = g_\theta = 0$$

also, since velocity is in the y direction only, we have

$$v_x = v_\theta = 0$$

Equation (1) becomes

$$\frac{\partial P}{\partial x} = 0 \Rightarrow P \neq f(x) \tag{4}$$

Equation (2) become

$$\frac{\partial P}{\partial \theta} = 0 \Rightarrow P \neq f(\theta) \tag{5}$$

Equation (3) becomes

$$\frac{\partial P}{\partial y} = \frac{\mu}{x} \frac{\partial}{\partial x}\left(x \frac{\partial v_y}{\partial x}\right) \tag{6}$$

Implies

$$\frac{\partial P}{\partial y} = \frac{\mu}{x} \frac{\partial}{\partial x}\left(x \frac{\partial v_y}{\partial x}\right) = \text{constant} \tag{7}$$

Since the L.H.S of equation (6) contains function of y only while the R.H.S contains function of x only. Hence, let the pressure at the 2 ends of the pipe be $P_1$and P_2. See Fig. 3
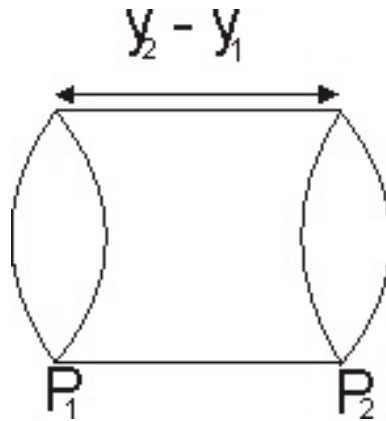

Fig. 3. Circular pipe with pressure P₁ and P₂ at its ends

To have flow in the y direction, $P_1 > P_2$. Let $P_1 - P_2 = \Delta P$ and $y_1 - y_2 = L$, then equation (7) becomes

$$\frac{dP}{dy} = \frac{P_2 - P_1}{y_2 - y_1} = \frac{-\Delta P}{L} = \frac{\mu}{x}\frac{d}{dx}\left(x\frac{dv_y}{dx}\right)$$

Implies,

$$\frac{-\Delta P}{L} = \frac{\mu}{x}\frac{d}{dx}\left(x\frac{dv_y}{dx}\right) \qquad (8)$$

Rearranging and integrating (8) we have

$$\int \frac{d}{dx}\left(x\frac{dv_y}{dx}\right)dx = -\int \frac{x}{\mu}\frac{\Delta P}{L}dx$$

Implies,

$$x\frac{dv_y}{dx} = -\frac{x^2}{2\mu}\frac{\Delta P}{L} + C_1 \qquad (9)$$

Integrating equation (9), we have

$$\int x\frac{dv_y}{dx}dx = -\int \left(\frac{x^2}{2\mu}\frac{\Delta P}{L} + C_1\right)dx$$

$$\int \frac{dv_y}{dx}dx = -\int \left(\frac{x}{2\mu}\frac{\Delta P}{L} + \frac{C_1}{x}\right)dx$$

Gives,

$$v_y = -\frac{x^2}{4\mu}\frac{\Delta P}{L} + C_1 lnx + C_2 \qquad (10)$$

To obtain the values of C_1 and C_2 in equation (10), we use the boundary conditions as follows
At $x = a$, $v_y = 0$, we have

$$0 = -\frac{a^2}{4\mu}\frac{\Delta P}{L} + C_1 lna + C_2$$

$$\Rightarrow C_2 = \frac{a^2}{4\mu}\frac{\Delta P}{L} - C_1 lna$$

And at $x = 0$, $v_y = $ finite, we have

$$v_y = C_1 \ln(0) + C_2$$

$$\Rightarrow C_1 = 0. \text{ (since } v_y \text{ is finite)}$$

Therefore:

$$v_y = -\frac{x^2}{4\mu}\frac{\Delta P}{L} + \frac{a^2}{4\mu}\frac{\Delta P}{L} = K\left(1 - \frac{x^2}{a^2}\right) \qquad (11)$$

Where:

$$K = \frac{a^2}{4\mu} \frac{\Delta P}{L}$$

Problem 2(Flow of Incompressible Fluid between two boundaries, one boundary is the air and the other boundary moving with a velocity, inclined at an angle θ):

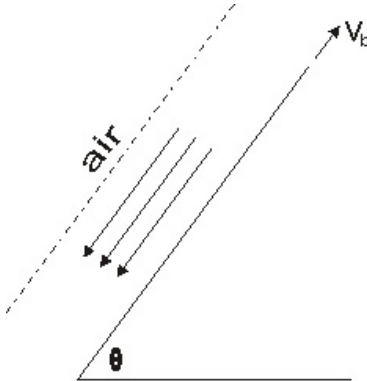Consider the flow of fluid bounded by two boundaries as shown in the Fig. 4

Fig. 4. Flow of Incompressible Fluid between two boundaries

One of the boundaries is the air while the other is moving with a velocity, v_b and the flow is downwards in the opposite direction to the moving boundary (P_{atm} is the pressure exerted by the atmosphere), See Fig. 5
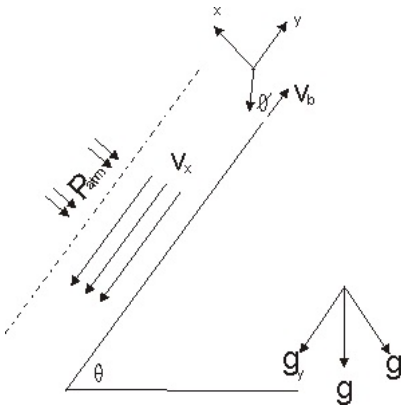

Fig. 5. Coordinate system of Flow of Incompressible Fluid between two boundaries

Assumptions:

1. Assume that the flow is in a steady state.
2. Assume that the fluid is incompressible (density is constant).
3. Assume that the fluid has a Newtonian behavior (viscosity is constant).
4. Assume that the fluid move strictly across the parallel boundaries.

Now applying the steps, we have

STEP 1:

The $x - y - \emptyset$ coordinate system is choosen where x, y and $\emptyset$ represent the parallel, perpendicular and orthogonal components (axes) respectively. Let

L = length of the pipe
v_x = velocity of the flow in the x direction
v_y = velocity of the flow in the y direction
$v_\emptyset$ = velocity of the flow in the $\emptyset$ direction
ρ = density of fluid (assumed constant)
μ = viscosity of fluid (assumed constant)
g_x = gravity acting on the fluid in the x direction
g_y = gravity acting on the fluid in the y direction
$g_\emptyset$ = gravity acting on the fluid in the $\emptyset$ direction

STEP 2:

Let the flow be driven by gravity

STEP 3:

The following are the boundary conditions:

At $x = L$, $\tau_y = 0$ where τ_y is the shear stress
At $x = 0$, $v_y = v_b$

Note that since velocity is in the y direction only, we have

$$g_x = -g\cos\theta$$
$$g_y = -g\sin\theta$$

STEP 4:

The Navier – Stokes equation (Conservation of momentum) for the coordinate system are as follows:[11]

For the x – axis

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_\emptyset\frac{\partial v_x}{\partial \emptyset}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial \emptyset^2}\right) \tag{12}$$

For the y – axis

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_\emptyset\frac{\partial v_y}{\partial \emptyset}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial \emptyset^2}\right) \tag{13}$$

For the θ – axis

$$\rho\left(\frac{\partial v_\emptyset}{\partial t} + v_x\frac{\partial v_\emptyset}{\partial x} + v_y\frac{\partial v_\emptyset}{\partial y} + v_\emptyset\frac{\partial v_\emptyset}{\partial \emptyset}\right) = -\frac{\partial P}{\partial \emptyset} + \rho g_\emptyset + \mu\left(\frac{\partial^2 v_\emptyset}{\partial x^2} + \frac{\partial^2 v_\emptyset}{\partial y^2} + \frac{\partial^2 v_\emptyset}{\partial \emptyset^2}\right) \tag{14}$$

At steady state

$$\frac{\partial v_x}{\partial t} = \frac{\partial v_y}{\partial t} = \frac{\partial v_\emptyset}{\partial t} = 0$$

then, equation (12) becomes

$$\frac{\partial P}{\partial x} = \rho g_x \tag{15}$$

Integrating equation (15) we obtain

$$P(x) = \rho g_x x + K_1 \tag{16}$$

But the pressure at the air boundary (i.e. where $x = L$) is the atmospheric pressure, P_{atm}, thus equation (16) becomes

$$P(L) = P_{atm} = -\rho L g cos\theta + K_1$$

$$K_1 = P_{atm} + \rho L g cos\theta$$

Therefore

$$P(x) = (L - x)\rho g cos\theta + P_{atm} \tag{17}$$

Equation (13) becomes

$$0 = -\frac{\partial P}{\partial y} + \rho g_y + \mu \frac{\partial^2 v_y}{\partial x^2}$$

$$\frac{\partial^2 v_y}{\partial x^2} = \frac{1}{\mu}\left(\frac{\partial P}{\partial y} - \rho g_y\right) \tag{18}$$

Integrating equation (18) twice we get

$$v_y = \frac{x^2}{2\mu}\left(\frac{\partial P}{\partial y} - \rho g_y\right) + K_2 x + K_1 \tag{19}$$

Now the shear stress

$$\tau_y = \mu\left(\frac{dv_x}{dy} + \frac{dv_y}{dx}\right) = \mu\frac{dv_y}{dx} \tag{20}$$

Since $v_x = 0$

Applying the boundary conditions at $x = 0$, $v_y = v_b$, hence from equation (19) we get

$$K_1 = v_b \tag{21}$$

Also at $x = L$, $\tau_y = 0$, using equations (19) and (20) we have

$$K_2 = -\frac{L}{\mu}\rho g sin\theta \tag{22}$$

Since from equation (17), P depends on x only, then $\frac{\partial P}{\partial y}$ is negligible.

Substituting equations (21) and (22) into equation (19), we obtain the velocity equation for the flow as follows

$$v_y = \frac{x^2}{2\mu}\left(\frac{\partial P}{\partial y} - \rho g_y\right) + K_2 x + K_1$$

$$v_y = \frac{x^2}{2\mu}(\rho g sin\theta) - \frac{L}{\mu}\rho x g sin\theta + v_b \tag{23}$$

3 Results and Discussion

In the problem 1, we considered a laminar flow through a pipe and found that the velocity profile was given by the equation

$$v_y = K\left(1 - \frac{x^2}{a^2}\right)$$

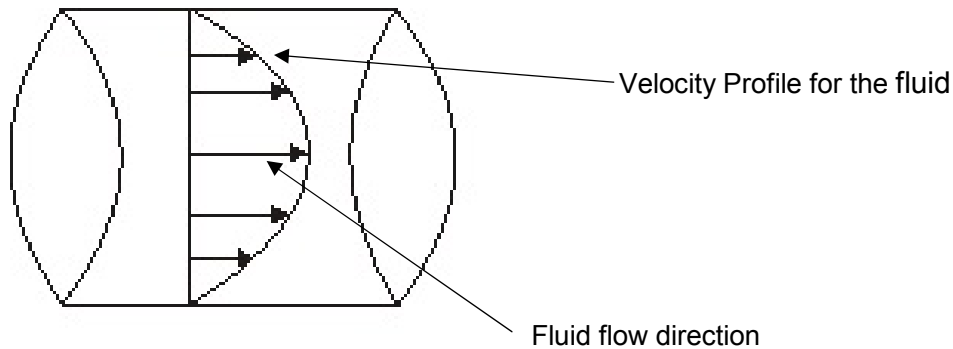Which is a parabola and thus the flow will look like the following diagram

Fig. 6. Velocity Profile for problem 1

Interpretation: The velocity profile from the above figure shows that for the flow problem i.e. laminar flow through a pipe, the flow moves in a parabolic shape through the pipe.

In the second problem, we considered flow of incompressible fluid between two boundaries, one boundary is the air and the other boundary moving with a velocity, inclined at an angle θ and found that the velocity profile was given by the equation

$$v_y = \frac{x^2}{2\mu}(\rho g sin\theta) - \frac{L}{\mu}\rho x g sin\theta + v_b$$

Which is a quadratic formula in the form

$$v_y(x) = Ax^2 + bx + c$$
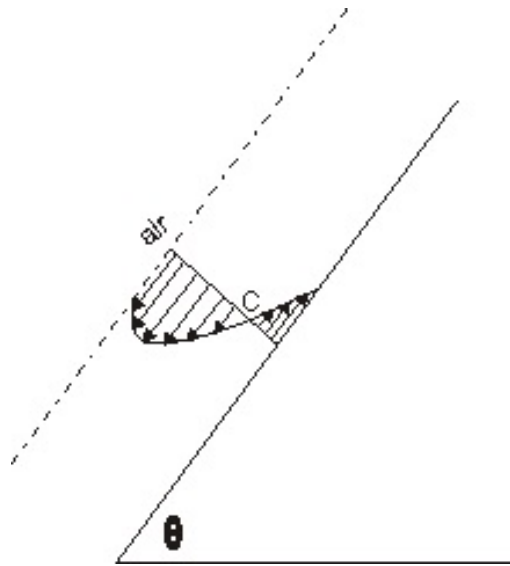
and thus the flow will look like the following diagram

Fig. 7. Velocity Profile for problem 2

The point C is where the velocity of the flow is 0 according to the intermediate value theorem.

Interpretation: The velocity profile of the flow between two boundaries, one boundary is the air and the other boundary moving with a velocity, inclined at an angle θ was found to follow a quadratic shape with a point C where the velocity was 0 (i.e. where the fluid is static and the flow direction changes).

4 Conclusion

The velocity profiles for the two different flow problems discussed were obtained. It was shown that for the first flow problem i.e. laminar flow through a pipe, the flow moves in a parabolic shape through the pipe while for the second problem in which the flow was between two boundaries, one boundary is the air and the other boundary moving with a velocity, inclined at an angle θ, the velocity profile was found to follow a quadratic shape with a point C where the velocity was 0. Since our approach and the results obtained in this study are not the same from the results obtained in the literature, which implies that the results of this paper are essentially new.

Competing Interests

Authors have declared that no competing interests exist.

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