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A coupled fixed point theorem for maps satisfying rational type contractive condition in dislocated quasi b-metric space

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Abstract: In this paper, a coupled fixed point theorem for maps satisfying rational type contractive condition in the perspective of dislocated quasi b-metric space have been formed and the existence and uniqueness of a coupled fixed point have been proved. Our result improves and generalizes comparable results in the literature.

Keywords: Continuous map, contractive condition, coupled fixed point, dislocated quasi *b*-metric space.

MSC: 47H06, 47H10, 54H25, 65D15.

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1. Introduction

In a complete dislocated metric space the celebrated Banach contraction principle was introduced and generalized by [1,2]. Since then, Zeyada [3] introduced the notion of dislocated quasi metric space for the first time. In this space, self-distance need not to be necessarily zero which is a very remarkable property of this space. After that the idea of dislocated quasi b-metric space was presented in [4].

The concept of coupled fixed point was extended by Guo & Lakshmikantham [5], where the monotone iterations technique is exploited. After that they introduced the concept for Partially ordered set. Bhaskar & Lakshmikantham [6] studied the existence and uniqueness of a coupled fixed point results in partially ordered metric space. They also introduced the concept of coupled fixed point and proved some fixed point theorems under certain contractive conditions. Moreover, after the work of Bhaskar & Lakshmikantham [6] coupled fixed point results were studied by many authors in different type of spaces [7]. Recently, Mohammed *et al.* [8] proved a coupled fixed point result in the setting of dislocated quasi-metric spaces.

Provoked by the result of Mohammed *et al.* [8], in this paper, a coupled fixed point theorem for maps satisfying rational type contractive condition in the perspective of dislocated quasi b-metric spaces have been established. Our result extends, improves and generalizes the comparable results in the existing literature. Moreover, we provided an example to support our main result.

2. Preliminaries

Throughout this paper, R^+ represents the set of non-negative real numbers and N represents the set of natural numbers. First, we recall some known definitions and lemmas.

Definition 1. [4] Let $X \neq \emptyset$ and $d : X \times X \to [0, \infty)$ be a function satisfying the conditions:

1. $d(x, y) = d(y, x) = 0 \Rightarrow x = y$,
2. $d(x, y) \leq k\,[d(x, z) + d(z, y)]$ for all $x, y, z \in X$.

Where $k \geq 1$ is any given real number. Then d is known as dislocated quasi b-metric on X and the pair (X, d) is called a dislocated quasi b -metric space or in short (dqb) metric spaces.

Definition 2. [4] A sequence $\{x_n\}$ in a dislocated quasi b-metric space (X, d) is said to converge to a point $x \in X$ if and only if

$$\lim_{n\to\infty} d(x_n, x) = 0 = \lim_{n\to\infty} d(x, x_n).$$

Definition 3. [4] A sequence $\{x_n\}$ in a dq b-metric space (X, d) is said be a Cauchy sequence if for each $\epsilon > 0$, there exists a positive integer n_0 such that for all $n, m \geq n_0$, we have $d(x_n, x_m) < \epsilon$. That is, $\lim_{n,m\to\infty} d(x_n, x_m) = 0$.

Definition 4. [4] A dislocated quasi b-metric space is called complete if every Cauchy sequence converges to an element in the same metric space.

Definition 5. [4] Let (X, d_1) and (Y, d_2) be two dislocated quasi b-metric spaces, then the mapping $T : X \to Y$ is said to be continuous if for each sequence $\{x_n\}$ which is convergent to x_0 in X, the sequence $\{Tx_n\}$ converges to Tx_0 in Y.

Lemma 1. *[4] In dislocated quasi b-metric space, the limit of a convergent sequence is unique.*

Definition 6. [9] Let $T : X \to X$ be a self-map in a complete metric space (X, d). Then T is said to be a contraction mapping if there exist a constant $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y) \qquad \text{for all } x, y \in X.$$

Definition 7. Let X be a nonempty set and $T : X \to X$ be a self-map. We say that x is a fixed point of T if $Tx = x$.

Theorem 1. *[10] Let $T : X \to X$ be a continuous contraction with $\lambda \in [0, 1)$ and $0 \leq \lambda < \frac{1}{k}$ for $k \geq 1$ in a complete dislocated quasi-b-metric space (X, d), then T has a unique fixed point in X.*

Definition 8. [6] Let X be any non-empty set. An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $T : X \to X$ if $T(x, y) = x$ and $T(y, x) = y$.

Theorem 2. *[8] Let (X, d) be a complete dislocated quasi-metric space and $T : X \to X$ be a continuous mapping satisfying the following rational type contractive conditions*

$$\begin{aligned} d[T(x,y), T(u,v)] \leq\ & a_1[d(x,u) + d(y,v)] + a_2[d(x, T(x,y)) + d(u, T(u,v))] \\ & + a_3[d(x, T(u,v)) + d(u, T(x,y))] + a_4\left[\frac{d(x,T(x,y))d(u,T(u,v))}{d(x,u)+d(y,v)}\right] \\ & + a_5\left[\frac{[d(x,u)+d(y,v)]}{[d(x,T(x,y))+d(u,T(u,v))]}[1+d(x,u)+d(y,v)]\right] \\ & + a_6\left[\frac{d(x,T(x,y))+d(x,T(u,v))}{1+d(u,T(u,v))d(u,T(x,y))}\right] \end{aligned}$$

for all $x, y, u, v \in X$ and all $a_1, a_2, a_3, a_4, a_5, a_6$ are non-negative constants with $2(a_1 + a_2 + a_5) + 4(a_3 + a_6) + a_4 < 1$. Then T has a unique coupled fixed point in $X \times X$.

3. Main Results

Theorem 3. *Let (X, d) be a complete dislocated quasi b-metric space with constant coefficient $k \geq 1$ and $T : X \times X \to X$ be a continuous map satisfying the following rational type contractive conditions:*

$$\begin{aligned} d[T(x,y), T(u,v)] \leq\ & a_1[d(x,u) + d(y,v)] + a_2[d(x, T(x,y)) + d(u, T(u,v))] + a_3[d(x, T(u,v)) + d(u, T(x,y))] \\ & + a_4\left[\frac{d(x,T(x,y))d(u,T(u,v))}{d(x,u)+d(y,v)}\right] + a_5\left[\frac{[d(x,u)+d(y,v)]}{[d(x,T(x,y))+d(u,T(u,v))]}[1+d(x,u)+d(y,v)]\right] \\ & + a_6\left[\frac{d(u,T(x,y))+d(x,T(u,v))}{1+d(u,T(u,v))d(u,T(x,y))}\right] + a_7\left[\frac{[d(y,v)+d(x,T(x,y))]d(u,T(u,v))}{1+d(y,v)+d(x,T(x,y))}\right] \end{aligned}$$

for all $x, y, u, v \in X$ and $a_1, a_2, a_3, a_4, a_5, a_6$, and a_7 are non-negative constants with $2ka_1 + (k + 1)(a_2 + a_5) + (2k^2 + 2k)(a_3 + a_6) + a_4 + a_7 < 1$. Then T has a unique coupled fixed point in $X \times X$.

Proof. Let x_0, $y_0 \in X$ be any two arbitrary points. We can construct two sequences $\{x_n\}$ and $\{y_n\} \in X$ such that $x_{n+1} = T(x_n, y_n)$ and $y_{n+1} = T(y_n, x_n)$ for $n = 0, 1, 2, \cdots$. Consider, $d(x_n, x_{n+1}) = d[T(x_{n-1}, y_{n-1}), T(x_n, y_n)]$.

Now, we have:

$$\begin{aligned} d(x_n, x_{n+1}) &= d[T(x_{n-1}, y_{n-1}), T(x_n, y_n)] \\ &\leq a_1[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + a_2[d(x_{n-1}, T(x_{n-1}, y_{n-1})) + d(x_n, T(x_n, y_n))] \\ &\quad + a_3[d(x_{n-1}, T(x_n, y_n)) + d(x_n, T(x_{n-1}, y_{n-1}))] + a_4 \left[\frac{d(x_{n-1}, T(x_{n-1}, y_{n-1}))d(x_n, T(x_n, y_n))}{d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} \right] \\ &\quad + a_5 \left[\frac{[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)]}{[d(x_{n-1}, T(x_{n-1}, y_{n-1})) + d(x_n, T(x_n, y_n))]} [1 + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] \right] \\ &\quad + a_6 \left[\frac{d(x_n, T(x_{n-1}, y_{n-1})) + d(x_{n-1}, T(x_n, y_n))}{1 + d(x_n, T(x_n, y_n))d(x_n, T(x_{n-1}, y_{n-1}))} \right] \\ &\quad + a_7 \left[\frac{[d(y_{n-1}, y_n) + d(x_{n-1}, T(x_{n-1}, y_{n-1}))]d(x_n, T(x_n, y_n))}{1 + d(y_{n-1}, y_n) + d(x_{n-1}, T(x_{n-1}, y_{n-1})} \right]. \end{aligned}$$

By using the definitions of the sequences $\{x_n\}$ and $\{y_n\}$, we have:

$$\begin{aligned} d(x_n, x_{n+1}) &\leq a_1[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + a_2[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \\ &\quad + a_3[d(x_{n-1}, x_{n+1}) + d(x_n, x_n)] + a_4 \left[\frac{d(x_{n-1}, x_n)d(x_n, x_{n+1})}{d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} \right] \\ &\quad + a_5 \left[\frac{[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)][d(x_{n-1}, x_n) + d(x_n, x_{n+1})]}{1 + d(x_{n-1}, x_n) + (y_{n-1}, y_n)]} \right] \\ &\quad + a_6 \left[\frac{d(x_n, x_n) + d(x_{n-1}, x_{n+1})}{1 + d(x_n, x_{n+1})d(x_n, x_n)} \right] + a_7 \left[\frac{[d(y_{n-1}, y_n) + d(x_{n-1}, x_n)]d(x_n, x_{n+1})}{1 + d(y_{n-1}, y_n) + d(x_{n-1}, x_n)} \right]. \end{aligned}$$

Using the triangle inequality and the fact that $d(x, y) \geq 0$, we have:

$$\begin{aligned} d(x_n, x_{n+1}) &= d[T(x_{n-1}, y_{n-1}), T(x_n, y_n)] \\ &\leq a_1[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] \\ &\quad + a_2[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] + ka_3[d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \\ &\quad + a_4 \left[\frac{d(x_{n-1}, x_n)d(x_n, x_{n+1})}{d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} \right] + a_5 \left[\frac{[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)][d(x_{n-1}, x_n) + d(x_n, x_{n+1})]}{1 + d(x_{n-1}, x_n) + d(y_{n-1}, y_n)} \right] \\ &\quad + ka_6 \left[\frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{1 + d(x_n, x_{n+1})d(x_n, x_{n+1})} \right] \\ &\quad + a_7 \left[\frac{[d(y_{n-1}, y_n) + d(x_{n-1}, x_n)]d(x_n, x_{n+1})}{1 + d(y_{n-1}, y_n) + d(x_{n-1}, x_n)} \right]. \end{aligned}$$

Simplifying the above inequality, we have:

$$\begin{aligned} d(x_n, x_{n+1}) &\leq a_1[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)] + a_2[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \\ &\quad + ka_3[d(x_{n-1}, x_n) + d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + d(x_n, x_{n+1})] \\ &\quad + a_4 d(x_n, x_{n+1}) + a_5[d(x_{n-1}, x_n) + d(x_n, x_{n+1})] + ka_6[d(x_{n-1}, x_n) \\ &\quad + d(x_n, x_{n+1}) + d(x_{n-1}, x_n) + d(x_n, x_{n+1})] + a_7 d(x_n, x_{n+1}). \end{aligned}$$

It follows that:

$$\begin{aligned} d(x_n, x_{n+1}) &\leq (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)d(x_n, x_{n+1}) \\ &\quad + (a_1 + a_2 + 2k(a_3 + a_6) + a_5)d(x_{n-1}, x_n) + a_1 d(y_{n-1}, y_n). \end{aligned}$$

Simplification yields:

$$\begin{aligned} d(x_n, x_{n+1}) \leq & \frac{a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(x_{n-1}, x_n) \\ & + \frac{a_1}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(y_{n-1}, y_n). \end{aligned} \tag{1}$$

Proceeding similarly, we can show that:

$$d(y_n, y_{n+1}) \leq \frac{a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(y_{n-1}, y_n) + \frac{a_1}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(x_{n-1}, x_n). \tag{2}$$

Adding (1) and (2), we get:

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq \frac{2a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} [d(x_{n-1}, x_n) + d(y_{n-1}, y_n)].$$

Since $2ka_1 + (k+1)(a_2 + a_5) + (2k^2 + 2k)(a_3 + a_6) + a_4 + a_7 < 1$ with

$$\lambda = \frac{2a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} < 1.$$

So, we have $k\lambda < 1$, thus, the above inequality becomes:

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq \lambda[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)].$$

Furthermore,

$$d(x_{n+1}, x_{n+2}) \leq \frac{a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(x_n, x_{n+1}) + \frac{a_1}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(y_n, y_{n+1}). \tag{3}$$

Similarly,

$$d(y_{n+1}, y_{n+2}) \leq \frac{a_1 + a_2 + 2k(a_3 + a_6) + a_5}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(y_n, y_{n+1}) + \frac{a_1}{1 - (a_2 + 2k(a_3 + a_6) + a_4 + a_5 + a_7)} d(x_n, x_{n+1}). \tag{4}$$

Adding (3) and (4), we obtain:

$$\begin{aligned} [d(x_{n+1}, x_{n+2}) + d(y_{n+1}, y_{n+2})] &\leq \lambda[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \\ &= \lambda^2[d(x_{n-1}, x_n) + d(y_{n-1}, y_n)]. \end{aligned}$$

Continuing this process inductively, we have:

$$[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] \leq \lambda^n[d(x_0, x_1) + d(y_0, y_1)].$$

Now, we show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in X. For non negative integers m and n with $m > n$, we have

$$\begin{aligned} [d(x_n, x_m) + d(y_n, y_m)] &\leq k[d(x_n, x_{n+1}) + d(x_{n+1}, x_m) + d(y_n, y_{n+1}) + d(y_{n+1}, y_m)] \\ &\leq k[d(x_n, x_{n+1}) + d(y_n, y_{n+1})] + k^2[d(x_{n+1}, x_{n+2}) + (d(y_{n+1}, y_{n+2})] \\ &\quad + ... + k^{m-n}[d(x_{m-1}, x_m) + d(y_{m-1}, y_m)] \\ &\leq k\lambda^n[d(x_0, x_1) + d(y_0, y_1)] + k^2\lambda^{n+1}[d(x_0, x_1) + d(y_0, y_1)] \\ &\quad + ... + k^{m-n}\lambda^{m-1}[d(x_0, x_1) + d(y_0, y_1)] \\ &= k\lambda^n(1 + k\lambda + (k\lambda)^2 + ... + (k\lambda)^{m-n-1})[d(x_0, x_1) + d(y_0, y_1)] \\ &\leq \frac{k\lambda^n}{1 - k\lambda}[d(x_0, x_1) + d(y_0, y_1)] \to 0 \text{ as } n \to \infty \end{aligned} \tag{5}$$

It follows that:

$$[d(x_n, x_m) + d(y_n, y_m)] \to 0 \quad as \quad n, m \to \infty$$

which in turn implies that:

$$d(x_n, x_m) \to 0 \text{ and } d(y_n, y_m) \to 0 \text{ as } n, m \to \infty.$$

Hence $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in a complete dislocated quasi b-metric space X. As a result there must exist $(x, y) \in X \times X$ such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$. In addition, Since T is continuous we have:

$$x = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} T(x_n, y_n) = T\Big(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\Big) = T(x, y).$$

Similarly,

$$y = \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} T(y_n, x_n) = T\Big(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n\Big) = T(y, x).$$

Therefore, $(x, y) \in X \times X$ is a coupled fixed point of T.

Uniqueness

Now, we show that $(x, y) \in X \times X$ is a unique coupled fixed point of T. Suppose that T has another different coupled fixed point say (x', y') where $(x', y') \in X \times X$ with $x' = T(x', y')$ and $y' = T(y', x')$. Then, by using (1), we have:

$$\begin{aligned}
d(x,x) &= d[T(x,y),T(x,y)]\\
&\leq a_1[d(x,x)+d(y,y)] + a_2[d(x,T(x,y))+d(x,T(x,y))] + a_3[d(x,T(x,y))+d(x,T(x,y))]\\
&\quad + a_4\left[\frac{d(x,T(x,y))d(x,T(x,y))}{d(x,x)+d(y,y)}\right] + a_5\left[\frac{[d(x,x)+d(y,y)][d(x,T(x,y))+d(x,T(x,y))]}{1+d(x,x_)+d(y,y)}\right]\\
&\quad + a_6\left[\frac{d(x,T(x,y))+d(x,T(x,y))}{1+d(x,T(x,y))d(x,T(x,y))}\right] + a_7\left[\frac{[d(y,y)+d(x,T(x,y))]d(x,T(x,y))}{1+d(y,y)+d(x,T(x,y))}\right].\\
&= a_1[d(x,x)+d(y,y)] + a_2[d(x,x)+d(x,x)] + a_3[d(x,x)+d(x,x)] + a_4\left[\frac{d(x,x)d(x,x)}{d(x,x)+d(y,y)}\right]\\
&\quad + a_5\left[\frac{[d(x,x)+d(y,y)][d(x,x)+d(x,x)]}{1+d(x,x_)+d(y,y)}\right] + a_6\left[\frac{d(x,x)+d(x,x)}{1+d(x,x)d(x,x)}\right]\\
&\quad + a_7\left[\frac{[d(y,y)+d(x,x)]d(x,x)}{1+d(y,y)+d(x,x)}\right].\\
&\leq a_1[d(x,x)+d(y,y)] + a_2[d(x,x)+d(x,x)] + a_3[d(x,x)+d(x,x)] + a_4 d(x,x)\\
&\quad + a_5[d(x,x)+d(x,x)] + a_6[d(x,x)+d(x,x)] + a_7 d(x,x).
\end{aligned}$$

Then simplification yields:

$$d(x,x) \leq \delta d(x,x) + a_1 d(y,y). \tag{6}$$

where $\delta = a_1 + 2(a_2 + a_3 + a_5 + a_6) + a_4 + a_7$.

In a similar way, we can confirm that:

$$d(y,y) \leq \delta d(y,y) + a_1 d(x,x). \tag{7}$$

Adding (6) and (7), we obtain:

$$[d(x,x) + d(y,y)] \leq \mu[d(x,x) + d(y,y)].$$

where $\mu = \delta + a_1$.

Since, $\mu < 1$ hence the above inequality is possible if and only if $d(x,x) + d(y,y) = 0$. Hence $d(x,x) = 0$ and $d(y,y) = 0$. Similarly, $d(x',x') = 0$ and $d(y',y') = 0$. Now, we consider:

$$\begin{aligned}
d(x,x') &= d[T(x,y),T(x',y')]\\
&\leq a_1[d(x,x')+d(y,y')] + a_2[d(x,T(x,y))+d(x',T(x',y'))]\\
&\quad + a_3[d(x,T(x',y'))+d(x',T(x,y))] + a_4\left[\frac{d(x,T(x,y))d(x',T(x',y'))}{d(x,x')+d(y,y')}\right]
\end{aligned}$$

$$+ a_5 \left[\frac{[d(x,x') + d(y,y')][d(x,T(x,y)) + d(x',T(x',y'))]}{1 + d(x,x') + d(y,y')} \right]$$
$$+ a_6 \left[\frac{d(x',T(x,y)) + d(x,T(x',y'))}{1 + d(x',T(x',y'))d(x',T(x,y))} \right] + a_7 \left[\frac{d(y,y') + d(x,T(x,y))d(x',T(x',y'))}{1 + d(y,y') + d(x',T(x',y'))} \right].$$

In fact $T(x,y) = x$ and $T(x',y') = x'$, then we have:

$$\begin{aligned} d(x,x') &= a_1[d(x,x') + d(y,y')] + a_2[d(x,x) + d(x',x')] + a_3[d(x,x') + d(x',x)] + a_4 \left[\frac{d(x,x)d(x',x')}{d(x,x') + d(y,y')} \right] \\ &+ a_5 \left[\frac{[d(x,x') + d(y,y')][d(x,x) + d(x',x')]}{1 + d(x,x') + d(y,y')} \right] + a_6 \left[\frac{d(x',x) + d(x,x')}{1 + d(x',x')d(x',x)} \right] \\ &+ a_7 \left[\frac{[d(y,y') + d(x,x')]d(x',x')}{1 + d(y,y') + d(x,x')} \right]. \end{aligned}$$

Since $d(x,x) = d(x',x') = 0$, we have:

$$\begin{aligned} d(x,x') &\leq (a_1 + a_3 + a_6)d(x,x') + (a_3 + a_6)d(x',x) + a_1 d(y,y') \\ (1 - (a_1 + a_3 + a_6))d(x,x') &\leq (a_3 + a_6)d(x',x) + a_1 d(y,y'). \end{aligned} \tag{8}$$

By following the similar procedure, we can get:

$$(1 - (a_1 + a_3 + a_6))d(y,y') \leq (a_3 + a_6)d(y',y) + a_1 d(x,x'). \tag{9}$$

Adding (8) and (9) and then simplifying, we obtain:

$$[d(x,x') + d(y,y')] \leq \sigma[d(x',x) + d(y',y)]. \tag{10}$$

where

$$\sigma = \left[\frac{a_3 + a_6}{1 - (2a_1 + a_3 + a_6)} \right].$$

Similarly, we can get:

$$[d(x',x) + d(y',y)] \leq \sigma[d(x,x') + d(y,y')]. \tag{11}$$

Adding (10) and (11), we get:

$$[d(x,x') + d(y,y') + d(x',x) + d(y',y)] \leq \sigma[d(x,x') + d(y,y') + d(x',x) + d(y',y)].$$

Since $\sigma < 1$ so, the above inequality is possible if and only if $[d(x,x') + d(y,y') + d(x',x) + d(y',y)] = 0$. Which implies that $d(x,x') = 0, d(y,y') = 0, d(x',x) = 0$, and $d(y',y) = 0$. It follows that $x = x'$ and $y = y'$ such that $(x,y) = (x',y')$ which contradicts our assumption. Therefore, (x,y) is a unique coupled fixed point of T in $X \times X$. □

Remark 1. If we take $k = 1$ and $a_7 = 0$ in Theorem 3, we get Theorem 2 of [8]. Thus our established theorem generalizes Theorem 2.

Example 1. Let $X = [0,1]$. Define $d : X \times X \to \Re^+$ by

$$d(x,y) = |2x + y|^2 + |2x - y|^2$$

for all $x,y \in X$. Then (X,d) is dq b-metric space with constant coefficient $k = 2$. If we define a continuous map $T : X \times X \to X$ by $T(x,y) = \frac{xy}{10}$. Since $|2xy + uv|^2 \leq |2x + u|^2 + |2y + v|^2, |2xy - uv|^2 < |2x - u|^2 + |2y - v|^2$ holds for all $x,y,u,v \in X$. Then, we have:
$d[T(x,y),T(u,v)] = d\left(\frac{2xy}{10}, \frac{uv}{10}\right) = \left|\frac{2xy}{10} + \frac{uv}{10}\right|^2 + \left|\frac{2xy}{10} - \frac{uv}{10}\right|^2 \leq \frac{1}{100}\left(|2x + u|^2 + |2y + v|^2 + |2x - u|^2 + |2y - v|^2\right)$
$= \frac{1}{10}\left[d(x,u) + d(y,v)\right].$

It shows that:

$$\begin{aligned} d[T(x,y),T(u,v)] \leq\ & a_1[d(x,u)+d(y,v)] + a_2[d(x,T(x,y))+d(u,T(u,v))] + a_3[d(x,T(u,v))+d(u,T(x,y))] \\ & + a_4\left[\frac{d(x,T(x,y))d(u,T(u,v))}{d(x,u)+d(y,v)}\right] + a_5\left[\frac{[d(x,u)+d(y,v)][d(x,T(x,y))+d(u,T(u,v))]}{1+d(x,u)+d(y,v)}\right] \\ & + a_6\left[\frac{d(u,T(x,y))+d(x,T(u,v))}{1+d(u,T(u,v))d(u,T(x,y))}\right] + a_7\left[\frac{[d(y,v)+d(x,T(x,y))]d(u,T(u,v))}{1+d(y,v)+d(x,T(x,y))}\right]. \end{aligned}$$

It can be easily shown that $(0,0) \in X \times X$ is the unique coupled fixed point of T in $X \times X$. Hence, we have successfully verified Theorem 3.

4. Conclusion and future scope

Mohammed *et al.* [8] proved the existence and uniqueness of a coupled fixed point result for maps satisfying certain rational type contractive condition in the setting of complete dislocated quasi metric space. In this paper, we have established and proved the existence and uniqueness of a coupled fixed point result for maps satisfying rational type contractive condition in the perspective of complete dislocated quasi b- metric spaces. Our established result generalizes and extends the result of Mohammed *et al.* [8] and related results in the existing literature. Also, we provided examples in support of the main result. The researchers believes that the search for the existence and uniqueness of coupled fixed point for maps satisfying different contractive conditions in dislocated quasi b- metric space is an active area of research. As a result, any interested researchers can utilize this opportunity to conduct their thesis work in this area.

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